[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 7115 **F-6** **Your Roll No...............**

Unique Paper Code : 2351603

Name of the Paper : Differential Equations – III

Name of the Course : **B.Sc. (Hons.) Mathematics (Erstwhile FYUP)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **three** sections in this question paper.
3. **Section A** is compulsory.
4. Attempt any **two** questions from each **Section B** and **C.**

SECTION A

(Attempt **all** *questions)*

1. Find the general solution in powers of x of the differential equation (10)

$$\left(x^2 - 4\right)y'' + 3xy' + y = 0$$

2. Use the method of Frobenius to find the Frobenius series solutions of the differential equation (10)

$$xy'' + 2y' + xy = 0.$$

3. Find the Fourier transform of

(a) $f(x) = \exp(-ax^2)$,

(b) $f(x) = \exp(-a|x|)$,

where a is a positive constant. (8)

P.T.O.

4. Show that the solution of the Dirichlet problem, if it exists, is unique. (7)

SECTION B

(Attempt any **two** *questions)*

5. Find the regular solution of Bessel's equation of first kind of order 0. (10)

6. State and prove the maximum principle (for a harmonic function on a bounded domain in the plane). (10)

7. State and prove Sturm Separation Theorem. Use it to show that between any two consecutive zeros of sin(2t) + cos(2t) there is precisely one zero of sin(2t) – cos(2t). (10)

SECTION C

(Attempt any **two** *questions)*

8. Find the solution of the Dirichlet problem in the half plane $y > 0$

$$u_{xx} + u_{yy} = 0, \quad -\infty < x < \infty, \quad y > 0$$

$$u(x,0) = f(x), \quad -\infty < x < \infty,$$

u and u_x vanish as $|x| \to \infty$, and u is bounded as $y \to \infty$. (10)

9. Define the convolution of the two functions f and g over $(-\infty, \infty)$. State and prove the convolution theorem. (10)

10. (a) Let f and g be two solution of

$$\frac{d}{dt}\left[P(t)\frac{dx}{dt}\right] + Q(t) = 0 \qquad (*)$$

such that f and g have a common zero on $a \le t \le b$. Show that f and g are linearly dependent on $a \le t \le b$.

(b) Let f and g be nontrivial linearly dependent solutions of equation (*) mentioned in part (a) on $a \le t \le b$. Suppose $f(t_0) = 0$, where $a \le$ to $\le b$. Show that $g(t_0) = 0$. (10)