17. Show that the function

$$f(x,y)=\frac{xy}{|xy|}$$

has no limit as (x, y) approaches $(0, 0)$.

18. If $w = \ln(x^2 + y^2 + z^2)$, $x = ue^v \sin u$, $y = ue^v \cos u$ and $z = ue^v$ find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ using chain rule at the point (u, v) = $(-2, 0)$.

19. Find the directions in which the function $f(x, y, z) = \ln xy + \ln yz + \ln xz$ increase and decrease most rapidly at the point $P_0(1, 1, 1)$. Then find the derivatives of the function in those directions.

20. Find parametric equations for the line tangent to the curve of intersection of the surfaces

$$x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0 \text{ and } x^3 + y^2 + z^2 = 11$$

at the point $(1, 1, 3)$.

21. Find the absolute maxima and minima of the function

$$T(x, y) = x^2 + xy + y^2 - 6x + 2$$

on the rectangular plate $0 \le x \le 5$; $-3 \le y \le 0$.

11/12/2018 (Evening)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 897 **I**

Unique Paper Code : 32355101

Name of the Paper : Calculus

Name of the Course : **Mathematics : G.E. for Honours**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Do any **five** questions from each of the **three** sections.
3. Each question is for **five** marks.

SECTION 1

1. Given $f(x) = 2x - 2$, $x_o = -2$, $\varepsilon = 0.02$. Find $L = \lim_{x \to x_o} f(x)$. Then find a number $\delta > 0$ such that for all x, $0 < |x - x_o| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

2. Find a linearization of $f(x) = \sqrt{x^2 + 9}$ at $x = -4$.

P.T.O.

3. The radius of a circle is increased from 2.00 to 2.02 m. Estimate the resulting change in area. Also express the estimate as a percentage of the circle's original area.

4. Evaluate the limit $\lim_{x\to 0} \frac{e^{1/x}-1}{e^{1/x}+1}$.

5. Use L'Hôpital's rule to find $\lim_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x}$.

6. Sketch the graph of a function $f(x) = x^3 - 3x + 1$.

7. Find the volume of the solid generated by revolving the region between the y-axis and curve $x = 2/y$, $1 \le y \le 4$, about the y-axis.

SECTION 2

8. Use the shell method to find the volume of the solid generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the y-axis.

9. Sketch the graph of $r = 1 - 2\cos\theta$ and identify its symmetries.

10. Find the area of the surface generated by revolving the curve $y = \sqrt{x}$, $0 \le x \le 1$, about the x-axis.

11. Suppose a person on a hang glider is spiraling upward due to rapidly rising air on a path having acceleration vector $a(t) = -3\cos t\,\boldsymbol{i} - 3\sin t\,\boldsymbol{j} + 2\boldsymbol{k}$. It is also known that initially (at time $t = 0$), the glider departed from the point (3,0,0) with velocity $v(0) = 3\boldsymbol{j}$. Find the glider's position as a function of t.

12. Find the unit tangent vector of the curve

$$r(t) = t^2\,\boldsymbol{i} + 2\cos t\,\boldsymbol{j} + 2\sin t\boldsymbol{k}.$$

13. Determine whether $\int_{-\infty}^{-1} \frac{1}{x}\,dx$ converges?

14. Find the arc length parameterization of the helix

$$r(t) = \cos 4t\,\boldsymbol{i} + \sin 4t\,\boldsymbol{j} + 3t\,\boldsymbol{k}, \quad 0 \le t \le 2\pi.$$

SECTION 3

15. Show that the ellipse $x = a\cos t$, $y = b\sin t$, $a > b > 0$, has its largest curvature on its major axis and its smallest curvature on its minor axis.

16. Find the binormal vector $\vec{B}$ and the torsion function τ for the space curve

$$\vec{r}(t) = (\cos t + t\sin t)\hat{i} + (\sin t - t\cos t)\hat{j} + 3\hat{k}$$

P.T.O.