25/11/16 (Morning) Friday

This question paper contains **4+1** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1331**

Unique Paper Code : **2341501** **F-7**

Name of the Paper : **Probability Theory and Statistical Computing**

Name of the Course : **B.Tech. Computer Science**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory. (7×5=35 marks)

Attempt any *four* questions from **2** to **7**. (2×5=10 marks each)

Parts of a question must be answered together.

Use of Non-Programmable Scientific Calculator is allowed.

The symbols have their usual meaning.

1. (*a*) State Baye's theorem. There are three coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the three coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin ?

(*b*) Derive Poisson distribution as a limiting case of Bionomial Distribution.

P.T.O.

(*c*) Let X be a random variable with probability density :

$$f(x) = \begin{cases} c\left(1 - x^2\right), & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(*i*) What is the value of c ?

(*ii*) What is the cumulative distribution function of X ?

(*d*) The joint density of X and Y is :

$$f(x, y) = \begin{cases} \dfrac{\left(y^2 - x^2\right)e^{-y}}{8}, & 0 < y < \infty, -y \le x \le y. \end{cases}$$

Show that $E[X|Y = y] = 0$.

(*e*) If X_1 and X_2 are independent binomial random variables with respective parameters (n_1, p) and (n_2, p), calculate the conditional probability mass function of X_1 given that $X_1 + X_2 = m$.

(*f*) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2. Transform the above process as Markov chain and write its transition probability matrix.