This question paper contains **4** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1415**

Unique Paper Code : **2341504** **F-7**

Name of the Paper : **Mathematical Physics—II**

Name of the Course : **B.Tech. (Computer Science) Allied Course**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do *Five* questions in all.

Question No. **1** is compulsory.

1. Do any *five* questions : 5×3=15

(*a*) Classify (order, degree, linear/non-linear) the following differential equation :

$$5\left(\frac{d^3y}{dx^3}\right)^4 + 3x^2\left(\frac{d^2y}{dx^2}\right)^5 + 4x\left(\frac{dy}{dx}\right)^7 + y = x^3.$$

(*b*) Check whether the following functions are linearly dependent or independent :

$$e^x \sin x,\ e^x \cos x.$$

(*c*) Prove the following property of Poisson Bracket :

$$[a,\ bc] = b[a,\ c] + [a,\ b]c.$$

(*d*) Find the extreme points of the function :

$$f(x, y) = 4xy - x^4 - y^4.$$

(*e*) Solve :

$$(x^2 + y^2)dx - 2xydy = 0.$$

(*f*) Define generalised momenta for *n*-particle system, and find its time derivative.

(*g*) Form the differential equation whose only solutions are :

$$a_1e^x, \; a_2e^{2x}, \; a_3e^{3x}.$$

(*h*) Find the extremal of the integral :

$$\int_0^{\pi} \left(y'^2 - y^2\right)dx, \text{ here } y' = \frac{dy}{dx}.$$

2. Solve the following differential equations :

(*a*) $x^4 \dfrac{dy}{dx} + x^3 y = -\sec\left(xy\right)$ 6

(*b*) $\dfrac{dy}{dx} = \dfrac{y + x - 2}{y - x - 4}.$ 9

3. Solve the following differential equations :

(*a*) $\dfrac{d^2 y}{dx^2} - 4\dfrac{dy}{dx} + 4y = e^{2x}\left(x^3 + \cos 3x\right)$ 6

(*b*) $x^2 \dfrac{d^2 y}{dx^2} - 3x\dfrac{dy}{dx} + 5y = x^2 \sin\left(\ln x\right).$ 9

4. (*a*) Solve the following differential equation : 6

$$(x^3 + y^3)dx - 2xy^2dy = 0.$$

(*b*) Using the method of variation of parameters, solve : 9

$$(D^2 + 9)y = x \cos 3x; \ D \equiv \frac{d}{dx}.$$

5. (*a*) Using the method of undetermined coefficients, solve : 6

$$(D^2 + 1)y = 2e^x + \cos x; \ D \equiv \frac{d}{dx}.$$

(*b*) Solve the coupled differential equations : 9

$$\frac{dx}{dt} + \frac{dy}{dt} - x = 2t + 1$$

$$2\left(\frac{dx}{dt} + \frac{dy}{dt}\right) + x = t.$$

6. (*a*) Prove that the equation of the shortest path between two points on the surface of right circular cylinder of radius a is given by :

$$z = c_1\phi + c_2,$$

where c_1 and c_2 are constants. 6

(*b*) Using Lagrange's method of undetermined multiplier, find the area of the largest triangle inscribed in the circle of radius a. 9

7. (*a*) Find the Hamiltonian corresponding to the Lagrangian : 6

$$L = a\dot{x}^2 + b\dot{y}^2 - kxy.$$

(*b*) Starting from the expression :

$$H(q, p) = p\dot{q} - L(q, \dot{q})$$

Derive the Hamilton's equations of motion. 9

8. (*a*) Show that the time derivative of $u(q, p)$ is given by :

$$\frac{du}{dt} = [u, H]$$

here, H denotes Hamiltonian. 6

(*b*) Write the Lagrangian of the system of two masses m and $2m$, shown below in Fig. 1. In this figure, x_1 and x_2 are the displacements of two masses from their equilibrium positions. Hence obtain the equations of motion of these two masses. 9

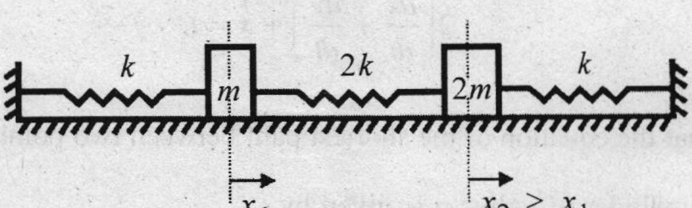

Fig. 1