6. (a) Show that for the distribution $f(x,\theta) = \theta e^{-\theta x}$; $0 < x < \infty$, central confidence limits for large samples with 95% confidence coefficient are given by:

$$\theta = \left(1 \pm \frac{1.96}{\sqrt{n}}\right) / \bar{x} .$$

(b) If $x \geq 1$ is the critical region for testing simple H_0 : $\theta = 2$ against the simple alternative $H_1 : \theta = 1$, on the basis of the single observation from the population:

$$f(x,\theta) = \theta e^{-\theta x} ; 0 < x < \infty ,$$

obtain the values of type I error, type II error and power of the test. 6,6

7. Write short notes on any *three* of the following:

(a) Rao-Blackwell theorem

(b) Efficient estimators

(c) Point estimation and Interval estimation

(d) Relation between t and chi square distribution. 4,4,4

11/12/18 (Evening)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **1334**	**I**
Unique Paper Code	: **62374311**	
Name of the Paper	: **Theory of Statistical Inference**	
Name of the Course	: **B.A. (Prog.) Statistics**	
Semester	: **III**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt six questions in all. Question No. 1 is compulsory. Attempt five more questions.

Use of simple calculator is allowed.

1. (a) Fill in the blanks:

(i) Variance of t distribution with $n-1$ d.f. is – .

(ii Karl Pearson's coefficient of skewness of chi-square distribution with n (> 2) d.f. is – .

(iii) Probability of type II error is denoted by – .

(iv) If $\overline{X}$ is a consistent estimator of θ then consistent estimator of $\theta(1-\theta)$ is – .

(v) If T is an unbiased estimator of θ then T^2 is a – estimator of θ^2 .

(vi) Sum of points of inflexion of F distribution with (n_1, n_2) d.f. is – . 1,1,1,1,1,1

P. T. O.

(b) Explain the following:

(i) Simple and composite hypotheses with a suitable example.

(ii) P-value.

(iii) Critical value, critical region and level of significance. 3,3,3

2. (a) Define consistency and explain its limitations. If X_1, X_2 ..., X_n are random observations on a Bernoulli variate X taking the value 1 with probability p and value 0 with probability $(1-p)$ then obtain a consistent estimator of $p(1-p)$.

(b) Show that $T = \frac{\sum x_i (n-\sum x_i)}{n(n-1)}$ is an unbiased estimator of $\theta(1-\theta)$ based on the random sample of size n from Bernoulli distribution with probability of success as θ. 6,6

3. (a) Define Student's t statistic and Fisher's t statistic. Show that Student's t may be regarded as a particular case of Fisher's t.

(b) The means of two large samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation 2.5 inches? (Test at 5% level of significance.) 6,6

4. (a) Define MVU estimator and show that MVU estimator is unique.

(b) Let X_1, X_2 ..., X_n be a random sample of size n from a population with pdf:

$$f(x,\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}; & 0 < x < \infty,\ \theta > 0 \\ 0 \quad ; & \text{otherwise} \end{cases}.$$

Find MVB estimator and maximum likelihood estimator of θ. 6,6

5. (a) Define chi-square statistic. Write the probability density function of chi-square distribution with n degrees of freedom. Obtain mean, variance and moment generating function of chi-square distribution with n degrees of freedom.

(b) Two sample polls of votes for two candidates A and B for a public office are taken from among the residents of rural and urban areas. The results are given in the following table:

Area	Votes for	
	A	B
Rural	620	380
Urban	550	450

Examine whether the nature of the area is related to voting preference in this election. (Given: the value of chi-square at 5% level of significance for 1 d.f. is 3.84) 6,6