8.(a) Obtain harmonic mean for beta distribution of second kind.

(b) If X ~Bin (n, p), find mode of the distribution. (6,6)

23|5|18 (Evening)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper : 8203 HC

Unique paper code : 62371201

Name of the paper : **Statistical Methodology**

Name of course : **B.A. (Prog.) Statistics**

Semester : **II**

Duration : **3 hours**

Maximum marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt six questions in all, including Q. No. 1 which is compulsory. Simple calculator can be used.

1.(a) A random variable X has the following probability function:

x	−2	−1	0	1	2	3
P(X=x)	0.1	k	0.2	$3k$	$2k$	0.3

Find k, $P(x<2)$, and $P(-2<x<2)$

(b) Let X denote the number of successes preceding the first failure. Find E(X).

(c) State whether the following statement is true or false: Mean of binomial distribution is 2 and variance is 6. Justify your answer.

(d) Find the r^{th} moment about origin for beta distribution of first kind.

(e) A Poisson variate X is such that P(X=2) = 9P(X=4) + 90P(X=6). Find its mean and variance. (3,3,3,3,3)

2.(a) Let X be a random variable with probability density function $f(x) = kx^2(1-x^3)$, $0 \leq x \leq 1$, where k is a constant. Find the value of k, mean and variance of X.

(b) If X is a binomial variate with parameters n and p, show that:

$$\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d\mu_r}{dp}\right] ;\ r = 1, 2, 3 \ldots \quad (6,6)$$

3.(a) Let X and Y be two random variables with the following joint probability mass function:

X / Y	-1	0	1
-1	0	0.1	0.1
0	0.2	0.2	0.2
1	0	0.1	0.1

(i)Find the variance of 2X.

(ii)Given that Y=0, what is the conditional probability distribution of X?

(b) Show that Poisson distribution is a limiting case of negative binomial distribution. (6,6)

4.(a) If X and Y are independent Poisson variates, show that the conditional distribution of X given X+Y, is binomial.

(b) State and prove Chebyshev's inequality. (6,6)

5.(a) Examine whether the weak law of large numbers holds for the sequence $\{X_k\}$ of independent random variables defined as follows:

$$P(X_k = \pm 2^k) = 2^{-(2k+1)},\ P(X_k = 0) = 1-2^{-2k}$$

(b) Define hypergeometric distribution. Obtain its mean and variance. (6,6)

6.(a) Let the random variable X follow $N(\mu, \sigma^2)$. Find its m.g.f. and hence deduce that all odd order central moments vanish and $\mu_{2n} = 1.3.5 \ldots (2n-1)\,\sigma^{2n}$.

(b) Prove that for the normal distribution, quartile deviation, mean deviation from mean and standard deviation respectively are approximately in the ratio 10 : 12 : 15. (6,6)

7.(a) If X is a Gamma Variate with mean n, find the m.g.f. of $z = \frac{X-n}{\sqrt{n}}$ and show that it approaches exp $(t^2/2)$ as n tends to infinity. Also interpret the result.

(b) Compute m.g.f. for exponential distribution. Hence, obtain its mean and variance. (6,6)