15/05/2018 (Evening)

[This question paper contains 7 printed pages]

Your Roll No. : .....

Sl. No. of Q. Paper : 9650 **HC**

Unique Paper Code : 62357602

Name of the Course : **B.A.(Programme) Mathematics : DSE-4**

Name of the Paper : Numerical Analysis

Semester : VI

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) All **six** questions are compulsory.

(c) Attempt any **two** parts from each question.

(d) Use of non-programmable scientific calculator is allowed.

1. (a) Perform three iterations of Secant method to obtain a root of the equation 6

$$x^2 - 7 = 0$$

with initial approximation $x_0 = 2,\ x_1 = 3$.

P.T.O.

(b) Perform four iterations of bisection method to obtain the root of the equation

$$x^3 - 3x^2 - 0.09x + 0.27 = 0$$ 6

in the interval $[0,1]$.

(c) If true value is 0.000310698 and approximated value is 0.0049065, then find the absolute and the relative error. Differentiate between round-off error and truncation error. 6

2. (a) Perform two iterations of Newton's method to solve the non-linear system of equations 6

$$x^2y + y^3 = 10$$

$$xy^2 - x^2 = 3$$

with initial approximation $(x_0, y_0) = (0.8, 2.2)$.

(b) Perform three iterations of Regula Falsi method to find the root of the equation

$$f(x) = x^3 - 2 = 0$$ 6

in the interval $[1,2]$.

(c) Apply finite difference method (second order) to solve the given problem : 6.5

$$y'' = y + x, \quad y(0) = 0, \quad y(1) = 0,$$

with $h = 1/4$.

(b) Find integral of $I = \int_0^2 e^{-x^2} dx$ using Simpson's rule and Trapezoidal rule. (6.5)

(c) Evaluate $\int_0^1 \frac{dx}{1+x}$ using composite Trapezoidal rule and Romberg integration with $h = 1$ and $h = 1/2$ only. 6.5

6. (a) Employ the classical fourth order RK method to integrate $y' = 4e^{0.8t} - 0.5y$, $y(0) = 2$ from $t = 0$ to 1 using a step size of 1. 6.5

(b) Apply Euler's method to approximate the solution of

$$y' = x + y, \quad y(0) = 2, \qquad 6.5$$

and calculate $y(1)$ by using h=0.5.

(c) Perform three iterations of Newton-Raphson method to obtain the cube root of 17 with initial approximation $x_0 = 2$. 6

3. (a) Find the inverse of the following matrix using the Gauss Jordan method : 6.5

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

(b) Solve the tridiagonal system $AX = b$ by using Gauss- Thomas algorithm.

$$\begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 3 \\ 0 & 3 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 17 \\ 22 \end{pmatrix}$$

6.5

(c) For the following system of equations: 6.5

$$10x + 4y - 2z = 12$$

$$x - 10y - z = -10$$

$$5x + 2y - 10z = -3$$

(i) Show that Jacobi iteration scheme converges.

(ii) Perform three iterations of Jacobi iteration scheme starting with initial vector $(x, y, z) = (0,0,0)$.

4. (a) For the following data, obtain the Gregory - Newton backward difference polynomial.

x	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.00	2.28

Also find the approximate value of f at $x = 0.25$. 6

(b) Prove the following :

$$1 + \Delta = 1 + \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}$$

(c) The function $f(x) = \sin(x)$ is defined on [1,3]. Find Lagrange interpolating linear polynomial on intervals [1, 2] and [2, 3]. Hence find the approximate value of $f(1.5)$ and $f(2.5)$. 6

5. (a) The following data for the function $f(x) = x^4$ is given (6.5)

X	0.4	0.6	0.8
f(x)	0.0256	0.1296	0.4096

Find $f'(0.8)$ and $f''(0.8)$ using quadratic interpolation.