(ii) $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^{1/n}$ **(6)**

(c) Prove that every monotonic function f on $[a, b]$ is Riemann integrable on $[a, b]$. **(6)**

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18/05/18 (Evening)

This question paper contains 4 printed pages. (Friday)

Your Roll No. ....

S. No. of Paper	: **8229**	**HC**
Unique paper code	: **62354443**	
Name of the paper	: **Analysis**	
Name of course	: **B.A. (Prog.) Mathematics**	
Semester	: **IV**	
Duration	: **3 hours**	
Maximum marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has ***six*** *questions in all.*

Attempt any ***two*** *parts from each question.*

All *questions are compulsory.*

Marks are indicated against each part of the questions.

1.(a) If **A** and **B** are non-empty bounded above subsets of $\mathbb{R}$ and **C={x+y | x∈A, y∈B}** then show that:

$$\textbf{Sup (C) = Sup (A) + Sup (B)} \qquad \textbf{(6.5)}$$

(b) Define neighborhood of a point, an open set and a closed set. Give an example of each of the following:

(i) A non-empty set which is a neighborhood of each of its points with the exception of one point.

(ii) A non-empty open set which is not an interval.

(iii) A non-empty closed set which is not an interval.

(iv) A non-empty set which is neither an open set nor a closed set. **(6.5)**

(c) Define limit point of a set. Find the limit points of **Z,** the set of integers and **Q,** the set of rational numbers. **(6.5)**

2.(a) Prove that the intersection of a finite number of open sets is an open set. Is the intersection of infinite family of open sets an open set? Justify. **(6.5)**

(b) Show that every continuous function on a closed interval is bounded. **(6.5)**

(c) Show that the function $f(x) = \sin\left(\frac{1}{x}\right)$ is not uniformly continuous on $]\,0, \infty[$. **(6.5)**

3.(a) When is a sequence $< a_n >$ said to converge to a number **a**? Show that the sequence $a_n = n^{1/n}, \forall n$ converges to **1** . **(6.5)**

(b) Prove that every monotonically increasing and bounded above sequence converges. **(6.5)**

(c) Show that $\lim_{n\to\infty}\left(\frac{n}{2^n}\right) = 0$ **(6.5)**

4.(a) Define a Cauchy sequence. Prove that a sequence of real numbers is convergent if and only if it is Cauchy. **(6)**

(b) Show that the sequence $< a_n >$ defined by

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}, \qquad \forall n$$

does not converge. **(6)**

(c) Show that the sequence $< a_n >$ defined by

$$a_1 = \frac{3}{2}, \quad a_{n+1} = 2 - \frac{1}{a_n}, \qquad \forall n \geq 1$$

is bounded and monotonic. Also find $\lim_{n\to\infty}(a_n)$. **(6)**

5.(a) State and prove Cauchy's general principle for the convergence of an infinite series. **(6)**

(b) Test for convergence the following series :

i. $\sum_{n=1}^{\infty}(\sqrt{n^4+1} - \sqrt{n^4-1})$

ii. $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$ **(6)**

(c) Test for convergence the series:

$$\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots,$$

for all positive values of x. **(6)**

6.(a) Prove that an absolutely convergent series is convergent. Give an example to show that the converse is not always true. **(6)**

(b) Test the convergence of the following series:

(i) $\sum_{n=1}^{\infty} \frac{(-1)^n \cos n\alpha}{n\sqrt{n}}$, α being real