(i) $\frac{1}{5}+\frac{\sqrt{2}}{7}+\frac{\sqrt{3}}{9}+\frac{\sqrt{4}}{11}+\cdots$ ... ...

(ii) $\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+\cdots$ ... ... **(6)**

6.(a) Let $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ be two series of positive terms such that $\lim_{n\to\infty}\frac{u_n}{v_n}=l$, where l is non-zero and finite. Prove that $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ converge or diverge together. **(6)**

(b) Define a conditionally convergent series. Test for convergence the series:

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots \text{ ... ...} \qquad \textbf{(6)}$$

(c) Prove that every continuous function f on $[a, b]$ is Riemann integrable on $[a, b]$.

(6)

17/05/2018 (Evening)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **8228**	**HC**
Unique paper code	: **62354443**	
Name of the paper	: **Analysis**	
Name of course	: **B.A. (Prog.) Mathematics**	
Semester	: **IV**	
Duration	: **3 hours**	
Maximum marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

*This question paper has **six** questions in all.*

*Attempt any **two** parts from each question.*

All questions are compulsory.

Marks are indicated against each part of the questions.

1.(a) When is a non-empty set of real numbers said to be bounded? Let **S** be a non-empty bounded subset of $\mathbb{R}$. Define **Sup(S)** and **Inf(S).** Write the supremum and infimum of the following sets:

(i) $\left\{1,\frac{1}{2},\frac{1}{2^2},\text{ ... ... }\frac{1}{2^n},\text{ ... ...}\right\}$

(ii) $\left\{-1,\frac{1}{2},-\frac{1}{3},\frac{1}{4},\text{ ... ... }\frac{(-1)^n}{n},\text{ ... ...}\right\}$ **(6.5)**

P. T. O.

17/05/2018 (Evening)

(b) If **A** and **B** are non-empty bounded below subsets of $\mathbb{R}$ and $C=\{x+y \mid x\in A, y\in B\}$ then show that:

$$\text{Inf}(C) = \text{Inf}(A) + \text{Inf}(B) \qquad (6.5)$$

(c) Define an open set. Prove that the union of an arbitrary family of open sets is an open set. (6.5)

2.(a) Define limit point of a set. Give an example of a set $S\subseteq\mathbb{R}$ which has:

(i) Infinite number of limit points

(ii) Exactly one limit point

(iii) Exactly two limit points. (6.5)

(b) Show that the function $f(x) = \sqrt{x}$ is uniformly continuous on $[1, 3]$. (6.5)

(c) Show that the function f defined on $\mathbb{R}$ by:

$$f(x) = \begin{cases} 1 & \textit{if } x \textit{ is rational} \\ 0 & \textit{if } x \textit{ is irrational} \end{cases}$$

is not continuous at any point of $\mathbb{R}$. (6.5)

3.(a) Show that a sequence can not converge to more than one limit. (6.5)

(b) Show that the sequence $< a_n >$ defined by:

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots \ldots\ldots\ldots + \frac{1}{n+n}, \quad \forall n$$

converges. (6.5)

(c) Prove that $\lim_{n\to\infty} n^{1/n} = 1$. (6.5)

4.(a) If $< a_n >$ and $< b_n >$ are sequences of real numbers such that $\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$ then prove that:

$$\lim_{n\to\infty}(a_n b_n) = ab \qquad (6)$$

(b) Let $< a_n >$ be a sequence defined by

$$a_1 = 1, \quad a_{n+1} = \frac{(2a_n + 3)}{4}, \forall\, n \geq 1,$$

Prove that $< a_n >$ is bounded above and monotonically increasing. Also find $\lim_{n\to\infty} a_n$. (6)

(c) Show that the sequence $< a_n >$ defined by

$$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{(2n-1)}, \forall n$$

does not converge. (6)

5.(a) If a series $\sum_{n=1}^{\infty} u_n$ is convergent then prove that :

$$\lim_{n\to\infty} u_n = 0.$$

Is the converse true? Justify. (6)

(b) Define the sum of a convergent series. Find the sum of the following series:

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \cdots \qquad (6)$$

(c) Test for convergence the following series: