(*c*) If α, β, γ are the roots of the equation :

$$x^3 + px^2 + qx + r = 0,$$

find the value of $\sum \frac{\beta^2 + \gamma^2}{\beta\gamma}$. 6½

5. (*a*) Find the multiplicative inverse of the following congruence classes :

(*i*) $[11]_{16}$ in Z_{16}

(*ii*) $[14]_{15}$ in Z_{15}. 6½

(*b*) Find the inverse and order of the following permutation :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 5 & 3 & 2 & 1 \end{pmatrix}$$

and construct its associated diagram. 6½

(*c*) Prove that the following set of matrices over R is a group under matrix multiplication : 6½

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}.$$

26/5/17 Friday (Evening)

This question paper contains **4+1** printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **2001**

Unique Paper Code : **62351201** **GC-4**

Name of the Paper : **Algebra**

Name of the Course : **B.A. (Prog.) Discipline Course**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Let V = F[*a*, *b*] be the set of all real valued functions defined on the interval [*a*, *b*]. For any *f* and *g* in V, *c* in **R**, we define :

$$(f \oplus g)\ (t) = f(t) + g(t),$$

$$(c \odot f)\ (t) = cf(t).$$

Prove the V is a vector space over **R**. 6

(*b*) Let V be the vector space $\mathbf{R}^3$ and let $v_1 = (1, 2, 1)$, $v_2 = (1, 0, 2)$, $v_3 = (1, 1, 0)$. Do v_1, v_2, v_3 span V ? Justify. 6

(*c*) Show that the set $S = \{v_1, v_2, v_3, v_4\}$ where

$v_1 = (1, 0, 1, 0)$, $v_2 = (0, 1, -1, 2)$, $v_3 = (0, 2, 2, 1)$ and $v_4 = (1, 0, 0, 1)$ is a basis for $\mathbf{R}^4$. 6

2. (*a*) Find the rank of the matrix

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 5 & -2 & 7 & 2 \\ 1 & 0 & 4 & -1 \\ 2 & 0 & 4 & -4 \end{bmatrix}$$

by reducing it to the normal form. 6½

(*b*) Solve the system of equations : 6½

$$x + 3y + 2z = 0$$

$$2x - y + 3z = 0$$

$$3x - 5y + 4z = 0$$

$$x + 17y + 4z = 0.$$

(*c*) Determine the eigenvectors of the matrix 6½

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

3. (*a*) If n is positive integer, prove that : 6

$$\left(1 + i\sqrt{3}\right)^n + \left(1 - i\sqrt{3}\right)^n = 2^{n+1} \cos\frac{n\pi}{4}.$$

(*b*) Prove that : 6

$$128 \cos^3 \theta \sin^5 \theta = \sin 8\theta - \sin 6\theta - 2 \sin 4\theta + 6 \sin 2\theta.$$

(*c*) Find all the values of $(1 + i)^{1/4}$. 6

4. (*a*) Form an equation whose roots are $1, -1, i, -i$. 6½

(*b*) If the sum of two roots of the equation :

$$4x^4 - 24x^3 + 31x^2 + 6x - 8 = 0$$

is zero, find all the roots of the equation. 6½

6. (*a*) Let $G = GL_2(R)$. Prove that :

$$D = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}, ad \neq 0 \right\}$$

is a subgroup of G. 6

(*b*) Prove that the set :

$S = \{f \in R : f(a) = 0\}$, where $a \in [0, 1]$

is a subring of R. 6

(*c*) Prove that the rigid motions of a square yields the subgroup of S_4. 6