28/11/16 (Eve)
Monday

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **1510**

Unique Paper Code : **2352601** **F-7**

Name of the Paper : **Numerical Methods**

Name of the Course : **Allied Paper (Erstwhile FYUP)**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

Use of Scientific Calculator is allowed.

1. (*a*) Define round-off error. Illustrate the effect of round-off error in adding numbers.

$$0.99 + 0.0044 + 0.0042$$

(*i*) Add from left to right, rounding to three digits at each step.

(*ii*) Add from right to left, rounding to three digits at each step.

(*iii*) Compare the relative error for parts (*i*) and (*ii*). 6

(*b*) Perform four iterations of bisection method to find the cube root of 2 in the interval (1, 2). 6

(*c*) Perform three iterations of Secant method to find the root of the equation :

$$x^3 + 3x^2 - 1 = 0$$

in the interval (0, 1). 6

2. (*a*) Use Newton's method to solve the non-linear system of equations :

$$f(x, y) = x^2 + y^2 - 1 = 0$$

$$g(x, y) = x^2 - y = 0$$

Take initial approximation as $(x_0, y_0) = (0.5, 0.5)$. Perform three iterations. 6.5

(*b*) Solve the followintg system of the equations by using Gaussian elimination (row pivoting) method. 6.5

$$x + 2y + 3z = 1$$

$$2x + 6y + 10z = 0$$

$$3x + 14y + 28z = -8$$

(*c*) Find the inverse of the following matrix using Gauss-Jordan method : 6.5

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

3. (*a*) Use Gauss-Seidel method to solve the given system of equations :

$$4x + y = 3$$

$$x + 3y - z = 4$$

$$-y + 4z = 5$$

Take initial approximation as $X^{(0)} = (0, 0, 0)^T$ and perform three iterations. 6

(*b*) Find the Lagrange interpolating polynomial from the following data :

$$x = [0, \quad 1, \quad 4, \quad 9]$$

$$f(x) = [0, \quad 1, \quad 2, \quad 3]$$

Hence, find an approximate value of $f(2)$. 6

(*c*) Prove that :

(*i*) $\Delta = (1 - \nabla)^{-1} - 1$

(*ii*) $\nabla = -\frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{\delta^2}{4}}$. 6

4. (*a*) For the following data, calculate the differences and obtain the forward difference polynomial. Interpolate at $x = 0.25$. 6.5

$$x = [0, \quad 0.1, \quad 0.2, \quad 0.3, \quad 0.4, \quad 0.5]$$

$$f(x) = [-1.5, \quad -1.27, \quad -0.98, \quad -0.63, \quad -0.22, \quad 0.25]$$

(*b*) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data :

$$x = [0.5, \quad 1.5, \quad 2.5]$$

$$f(x) = [0.125, \quad 3.375, \quad 15.625]$$

Hence, estimate the values of $f(1.0)$ and $f(2.0)$. 6.5

(*c*) Approximate the derivative of $f(x) = 1 + x + x^2$ at $x_0 = 1$, using the formula :

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0)}{h}$$

Take $h = 1, 0.1, 0.01$ and 0.001. 6.5

5. (*a*) Define degree of precision of a Quadrature rule. Approximate the value of the integral :

$$\int_{-1}^{1} \frac{1}{1+x^2}\, dx$$

using Trapezoidal rule. 6

(*b*) Approximate the value of the integral :

$$\int_{1}^{2} \frac{1}{x} dx$$

using Midpoint rule and Simpson's rule. 6

(*c*) Apply Euler's method to approximate the solution of the initial value problem :

$$\frac{dy}{dt} = y\, t^3 - 1.5y, \quad y(0) = 1$$

with $h = 1$ in the interval [0, 4]. 6

6. (*a*) Solve the initial value problem :

$$\frac{dy}{dx} = xy, \ \ y(0) = 1, \ 0 \le x \le 1$$

using Midpoint method with $h = 0.2$. 6.5

(*b*) Solve the initial value problem :

$$\frac{dy}{dx} = -2xy, \ \ 0 \le x \le 1, \ \ y(0) = 2, \ h = 0.5$$

using classical 4th order Runge-Kutta method. 6.5

(*c*) Apply finite-difference method to solve the problem :

$$\frac{d^2y}{dx^2} = y + x, \ \ 0 \le x \le 1, \ \ y(0) = 2, \ y(1) = 2.5$$

with $h = 0.25$. 6.5