Sl. No. of Ques. Paper : 6015

Unique Paper Code : 2352601

Name of Paper : **Numerical Methods**

Name of Course : **Allied ~~Paper (Erstwhile FYUP)~~** Course

F-9

Semester : **III / ~~V~~**

Duration : **3 Hours**

Maximum Marks : **75**

Instructions for candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Use of Scientific Calculator is allowed.

Q1. (a) Define round-off error. Illustrate the effect of round-off in the quadratic formula

$$x^2 - 57x + 1 = 0$$

(i) Use the standard quadratic formula, rounding to four digits.

(ii) Use the rationalized-numerator quadratic formula, rounding to four digits.

(iii) Compare the results from parts (i) and (ii) with the results found without rounding. (6)

(b) Perform four iterations of bisection method to find the cube root of 6 in the interval (1, 2). (6)

(c) Perform three iterations of Regula Falsi method to find the root of the equation

$$x^3 - 5x + 1 = 0$$

in the interval (0, 1). (6)

Q2 (a) Use Newton's method to solve the non-linear system of equations

$$f(x,y) = x^2 + 4y^2 - 16 = 0$$
$$g(x,y) = xy^2 - 4 = 0$$

Take initial approximation as $(x_0, y_0) = (1,1)$. Perform three iterations. (6.5)

(b) Solve the following system of the equations by using Gaussian elimination (row pivoting) method.

$$6x + 2y + 2z = 0$$
$$6x + 2y + z = 5$$
$$x + 2y - z = 0$$
(6.5)

(c) Find the inverse of the following matrix using Gauss-Jordan method

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 7 & 13 \\ 3 & 11 & 22 \end{bmatrix}$$
(6.5)

Q3. (a) Use Jacobi method to solve the given system of equations:

$$2x - y + z = -1$$
$$x + 2y - z = 6$$
$$x - y + 2z = -3$$

Take initial approximation as $X^{(0)} = (0,0,0)^T$ and perform three iterations. (6)

(b) Find the Lagrange interpolating polynomial from the following data :

$$x = [0, \quad 1, \quad 3]$$
$$f(x) = [1, \quad 3, \quad 55]$$

Hence, find an approximate value of $f(2)$. (6)

(c) Prove that :

(i) $\delta = \nabla(1 - \nabla)^{-1/2}$

(ii) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{\delta^2}{4}}$ (6)

Q4. (a) For the following data, calculate the differences and obtain the forward difference polynomial. Interpolate at $x = 0.35$.

$$x = [0.1, \quad 0.2, \quad 0.3, \quad 0.4, \quad 0.5]$$
$$f(x) = [1.40, \quad 1.56, \quad 1.76 \quad 2.00, \quad 2.28]. \quad (6.5)$$

(b) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data

$$x = [1, \quad 2, \quad 4, \quad 8]$$
$$f(x) = [3, \quad 7, \quad 21, \quad 73]$$

Hence, estimate the values of $f(3)$ and $f(7)$. (6.5)

(c) Approximate the derivative of $f(x) = 1 + x^2$ at $x_0 = 1$, using the formula :

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0)}{h}$$

Take $h = 1,\ 0.1,\ 0.01\ and\ 0.001$. (6.5)

Q5. (a) Define degree of precision of a Quadrature rule. Derive Trapezoidal rule for the evaluation of :

$$\int_a^b f(x)dx$$

Verify that it has degree of precision one. (6)

(b) Approximate the value of the integral

$$\int_{-1}^{1} \frac{1}{1 + x^2} dx$$

using Simpson's rule and Trapezoidal rule. (6)

(c) Apply Euler's method to approximate the solution of the initial value problem :

$$\frac{dy}{dt} = t + y, \quad y(0) = 2$$

with h=0.5 in the interval [0, 3]. (6)

Q6. (a) Solve the initial value problem :

$$\frac{dy}{dx} = -2xy^2, \ y(0) = 1, \ 0 \leq x \leq 1$$

using Midpoint method with h=0.2. (6.5)

(b) Solve the initial value problem :

$$\frac{dy}{dx} = y, 0 \leq x \leq 1, \ y(0) = 2, \ h = 0.5$$

using classical 4th order Runge-Kutta method. (6.5)

(c) Apply finite-difference method to solve the problem :

$$\frac{d^2y}{dx^2} = y + x(x-4), \ 0 \leq x \leq 4, \ y(0) = 0, \ y(4) = 0 .$$

with h=1. (6.5)