8/12/2016 (M)

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **1391**

Unique Paper Code : **2371504** **F-7**

Name of the Paper : **Stochastic Processes**

Name of the Course : **B.Sc. (Hons.) Statistics Erstwhile FYUP**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all,

selecting any *three* questions from each section.

All questions carry equal marks.

Use of simple calculator is allowed.

Section-I

1. (*a*) Let X be a non-negative integral valued random variable with probability distribution $P(X = n) = p_n$, $n = 0, 1, 2, \ldots$ and probability generating function $P(s) = \sum_{k=0}^{\infty} p_k s^k$. Obtain the generating functions for the following :

(*i*) $P(X > n)$,

(*ii*) $P(X = 2n)$.

(*b*) Let X be a random variable denoting the number of tosses required to get two consecutive heads when a fair coin is tossed. Find the probability generating function of X. Hence, find the expected number of trials needed. 6,6½

P.T.O.

2. (*a*) Let $S_N = X_1 + X_2 + ... + X_N$, where N has Poisson distrubution with mean a. If X_i's have i.i.d. Bernoulli distribution with $P[X_i = 1] = p$ and $P[X_i = 0] = 1 - p = q$, show that :

(*i*) S_N has Poisson distribution with mean ap.

(*ii*) the joint distribution of S_N and N has the probability mass function

$$P[N = n, S_N = y] = \frac{e^{-a} a^n p^y q^{n-y}}{y!(n-y)!}$$

for $n = 0, 1, 2, ...; y = 0, 1, 2 ..., n$ and the conditional distribution of N given $S_N = y$ has probability mass function

$$P[N = n | S_N = y] = \frac{e^{-aq}(aq)^{n-y}}{(n-y)!}, n \geq y.$$

(*iii*) $cov(N, S_N) = ap$.

(*b*) Define convolution. Let $X_1, X_2, ..., X_r$ be i.i.d. geometrically distributed random variables with mean q/p. Show that the convolution of X_i's, $i = 1, 2, ..., r$ is Negative Binomial.

6½, 6

3. (*a*) Define the following :

(*i*) Covariance Stationary

(*ii*) Gaussian Process

(*iii*) Markov Chain

(*iv*) Closed set

(*v*) Irreducible Markov Chain

(*vi*) Ergodic state.

(*b*) Let $\{X_n, n \geq 0\}$ be a Markov Chain having state space S = {1, 2, 3, 4} and transition matrix

$$P = \begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

Discuss the nature of the states. 6,6½

4. (*a*) Consider a sequence of random variables X_n, $n = 0, 1, 2, \ldots$ such that each of X_n assumes only two values -1 and 1 with conditional probabilities

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, 0 < a, b < 1.$$

Let $p_n = P(X_n = 1)$, $q_n = P(X_n = -1) = 1 - p_n$ for $n = 1, 2, \ldots$. Further, let $P(X_0 = 1) = p_1 = 1 - P(X_0 = -1)$ be the initial distribution. Find p_n, $E[X_n]$, and correlation coefficient between X_n and X_{n-1}.

(*b*) A computer device can be either in a busy mode (state 1) processing a task, or in an idle mode (state 2), when there are no tasks to process. Being in a busy mode, it can finish a task and enter an idle mode any minute with probability 0.2. Being in an idle mode, it receives a new task any minute with probability 0.1 and enters a busy mode. The initial state is given to be in idle mode. Let X_n be the state of the device after n minutes. Find : 6½, 6

(*i*) the distribution of X_1,

(*ii*) the steady state distribution of X_n,

(*iii*) the limiting distribution P^n as $n \to \infty$.

Section-II

5. (*a*) In the classical ruin problem, find an expression for the expected duration of the game.

(*b*) Write down the differential-difference equations for the Yule-Furry process. Obtain an expression for the size distribution $p_n(t) = P_r[N(t) = n]$, assuming that the process starts with one member at time $t = 0$. Further, obtain the probability generating function of $\{p_n(t)\}$. 6½, 6

6. (*a*) Show that for the linear growth process, the second moment $M_2(t)$ satisfies the differential equation $M'_2(t) = 2(\lambda - \mu)\, M_2(t) + (\lambda + \mu)\, M(t)$. Further, show that variance is

$$Var\ [X(t)] = \frac{i(\lambda + \mu)}{(\lambda - \mu)} e^{(\lambda-\mu)t}\left(e^{(\lambda-\mu)t} - 1\right); \lambda \neq u$$

where i is the population size at $t = 0$.

(*b*) At a one man barber shop, customers arrive according to Poisson distribution with a mean arrival rate of 5 per hour and his haircutting time was exponentially distributed with an average haircut taking 10 minutes. It is assumed that because of his excellent reputation, customers were always willing to wait. Calculate : 6½, 6

(*i*) The average number of customers in the shop and the average number of customers waiting for a haircut,

(*ii*) The percentage of time an arrival can walk right-in without having to wait,

(*iii*) The percentage of customers who have to wait prior to getting into the barber's chair.

7. (*a*) In the classical ruin problem, what will be the effect of reducing the uni stake from a dollar to half a dollar, on the probability of ruin of the gambler ?

(*b*) If $\{N(t)\}$ is a Poisson process, then show that the autocorrelation coefficient between $N(t)$ and $N(t+s)$ is $\sqrt{\frac{t}{t+s}}$.

(*c*) Show that the interval between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ having parameter λ has a negative exponential distribution. 4½, 4,4

8. (*a*) In the case of (M/M/1) : (∞/FIFO) queuing model, obtain the expressions for :

   (*i*) Expected number of customers in the system,

   (*ii*) The probability that the number of customers is $\geq k$,

   (*iii*) Variance of the queue length.

   (*b*) Divide the interval $[0, t]$ into n (a large number) intervals of equal length h and suppose that in each small interval, Bernoulli trials with probability of success λh and probability of failure $(1 - \lambda h)$ are held. Show that the number of successes in an interval of length t is a Poisson process with mean λt. State the assumptions you make. 6½, 6