4. (*a*) Derive the expression for the standard error of :

(*i*) the mean of a random sample of size n.

(*ii*) the difference of the means of two independent random samples of size n_1 and n_2.

(*b*) P_1 and P_2 are the (unknown) proportions of students wearing glasses in two universities A and B. To compare P_1 and P_2, samples of size n_1 and n_2 are taken from the two populations and the number of students wearing glasses is found to be x_1 and x_2 respectively. Suggest an unbiased estimate of $P_1 - P_2$ and obtain its sampling distribution when n_1 and n_2 are large. Hence explain how to test the hypothesis $H_0 : P_1 = P_2$ against $H_1 : P_1 \neq P_2$. 6,6

Section B

5. (*a*) Obtain mean deviation about mean of *t*-distribution with n d.f.

(*b*) If X is a Chi-square variate with n *df*, then prove that for large n :

$$\sqrt{2X} \sim N(\sqrt{2n}, 1)$$

This question paper contains **4+2** printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **6713**

Unique Paper Code : **32371301** HC

Name of the Paper : **Sampling Distribution**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory.

Attempt *six* questions in all by selecting at least *two* questions from each section.

1. Attempt any *five* parts : 5×3=15

(*a*) Define convergence in distribution and convergence in probability and state their relations.

(*b*) Discuss type-I and type-II errors and level of significance with examples.

(*c*) Decide whether the central limit theorem holds for the sequence of independent random variables X_k with distribution defined as $P(X_k = \pm k^{\alpha}) = 1/2$.

P.T.O.

(*d*) Show that the sum of independent Chi-square variates is also a χ^2 variate.

(*e*) If $X \sim F_{2,4}$, then show that :

$$P(X \geq 2) = 1/4.$$

(*f*) If $X \sim F_{m,n}$ and $Y \sim F_{n,m}$, then show that :

$P(X \leq a) + P(X \leq 1/a) = 1$ for all a.

(g) In a 2 × 3 contingency table, if $N = x + y + z$, $N' = x' + y' + z'$ and $N = N'$ then show that :

$$\chi^2 = \frac{(x - x')^2}{x + x'} + \frac{(y - y')^2}{y + y'} + \frac{(z - z')^2}{z + z'} \sim \chi^2_2.$$ 5×3

Section A

2. (*a*) If X is a random variable and $E(X^2) < \infty$, then prove that $P(|x| \geq a) \leq E(X^2)/a^2$, for all $a > 0$. Use Chebychev's inequality to show that for $n > 36$ the probability that in n throws of a fair die, the number of sixes lies between $\frac{n}{6} - \sqrt{n}$ and $\frac{n}{6} + \sqrt{n}$ is at least 31/36.

(*b*) If $X_1, X_2, \ldots, X_n$ are iid random variables with mean μ_1 and variance σ_1^2 (finite) and $S_n = X_1 + X_2 + \ldots + X_n$, then :

$$\lim_{n\to\infty} P\left[a \leq \frac{S_n - n\mu_1}{\sigma_1\sqrt{n}} \leq b\right] = \varphi(b) - \varphi(a), \text{ for } -\infty < a < b < \infty,$$

where $\varphi(.)$ is the distribution function of a standard normal variate. 6,6

3. (*a*) Let $\{X_n\}$ be a sequence of mutually independent random variables such that $P(X_n = \pm 1) = \frac{1 - 2^{-n}}{2}$ and $P(X_n = 0) = 2^{-n}$. Examine whether the weak law of large numbers can be applied to the sequence $\{X_n\}$.

(*b*) Given a random sample of size n from exponential distribution :

$$f(x) = \alpha e^{-\alpha x}, x \geq 0, \alpha > 0.$$

Show that $X_{(r)}$ and $W_{rs} = X_{(s)} - X_{(r)}$, $r < s$, are independent. Also find the distribution of $X_{(r+1)} - X_{(r)}$.

6,6

(c) Show that t-distribution tends to normal distribution for large n. 4,4,4

6. (a) For a Chi-square distribution with n d.f., prove that :

$$\mu_{r+1} = 2r(\mu_r + n\mu_{r-1}), r \geq 1.$$

Hence find β_1 and β_2. Also discuss the limiting form of χ^2 distribution.

(b) If $X \sim F_{m,n}$ distribution, obtain the distribution of mX when $n \to \infty$. Also obtain the mode of the F-distribution. 6,6

7. (a) Prove that if $n_1 = n_2$, the median of F-distribution is at $F = 1$ and that the quartiles Q_1 and Q_3 satisfy the condition $Q_1Q_3 = 1$.

(b) Discuss the t-test for testing the significance for the difference of two population means. 6,6

8. (a) Let $X_1, X_2, \ldots\ldots\ldots\ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ and $\bar{X}$ and S^2 respectively be the sample mean and sample variance. Let $X_{n+1} \sim N(\mu, \sigma^2)$, and

assume that $X_1, X_2, \ldots\ldots, X_n, X_{n+1}$ are independent. Obtain the sampling distribution of :

$$U = \frac{x_{n+1} - \bar{X}}{S}\sqrt{\frac{n}{n+1}}.$$

(b) If $X \sim F_{n_1, n_2}$, then show that its mean is independent of n_1.

(c) If X is Poisson variate with parameter λ and χ^2 is a Chi-square variate with 2K d.f., then prove that for all positive integers k :

$$P(X \leq k - 1) = P(\chi^2 > 2\lambda).$$ 4,4,4