(*b*) Evaluate

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}.$$

Section II

5. (*a*) Solve the differential equation :

$$\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2.$$

(*b*) Solve :

$$x^2 y\,dx - (x^3 + y^3)\,dy = 0.$$

6. Solve any *two* of the following differential equations :

(*a*) $(D^3 + 1)\,y = \cos 2x$

(*b*) $\frac{d^2y}{dx^2} + 4y = x\sin x$

(*c*) $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right).$

7. Solve the following partial differential equations :

(*a*) (*i*) $xz(z^2 + xy)\,p - yz(z^2 + xy)\,q = x^4.$

(*ii*) $(p^2 + q^2)\,y = qz.$

(*b*) $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y.$

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This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6712**

Unique Paper Code : **32371109** **HC**

Name of the Paper : **Calculus**

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory.

Attempt *four* more questions by selecting *two* questions from each Section.

1. Attempt any *five* parts :

(*a*) If

$$f(x) = \begin{cases} x+2 & ; \quad x < 1 \\ 4x - 1 & ; \quad 1 \le x \le 3 \\ x^2 + 5 & ; \quad x \ge 3 \end{cases}$$

Examine whether $\lim_{x\to 1} f(x)$ and $\lim_{x\to 3} f(x)$ exist.

(*b*) Evaluate

$$\lim_{x\to\infty}\left[\sqrt{x^2+x+1}-\sqrt{x^2+1}\right].$$

(*c*) Show that :

$$\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx.$$

(*d*) Evaluate the double integral :

$$\int_0^1\int_0^1 \frac{dxdy}{\sqrt{(1-x^2)}\sqrt{(1-y^2)}}.$$

(*e*) Solve

$$x\frac{dy}{dx}+y=y^2\log x.$$

(*f*) Solve the differential equation :

$$(D^2+D+1)y=\sin 2x.$$

(*g*) Form a partial differential equation by elimination of the constants h and k from :

$$(x+h)^2+(y-k)^2+z^2=c^2.$$

(*h*) Solve :

$$y^2p-xyq=x(z-2y).$$

Section I

2. (*a*) A function $f(x)$ is defined as follows :

$$f(x)=\begin{cases}\dfrac{x^2}{a}-a & ; \quad x<a\\ 0 & ; \quad x=a\\ a-\dfrac{a^2}{x} & ; \quad x>a\end{cases}$$

$a\neq 0$. Show that $f(x)$ is continuous at $x = a$.

(*b*) Find the value of the nth derivative of the function

$y=e^{m\sin^{-1}x}$, for $x = 0$.

3. (*a*) If $\theta=t^n e^{-r^2/4t}$, find the value of n which will make :

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)=\frac{\partial\theta}{\partial t}.$$

(*b*) Find the position and nature of the double points on the curve

$$y^3 = x^3 + ax^2.$$

4. (*a*) Show that :

$$\int_0^{\pi/2}\cos^{2m-1}\theta\sin^{2n-1}\theta\,d\theta=\frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}.$$