12. Let D be the region in the xy-plane that is bounded by the co-ordinate axes and the line $x + y = 1$. Use the suitable change of variable to compute the integral :

$$\iint_D \left(\frac{x-y}{x+y}\right)^6 dy\,dx.$$

Section III

13. State Green's theorem for simply connected regions. Use Green's theorem to find the work done by the force field $\mathbf{F}(x, y) = (e^x - y^3)i + (\cos y + x^3)j$ along the circle $x^2 + y^2 = 1$ in anticlockwise direction.

14. Give the geometrical interpretation of the surface integral $\iint ds$ over piecewise smooth surface S. Evaluate the surface integral $\iint xz\,ds$ over the surface S which is the part of the plane $x + y + z = 1$ that lies in the first octant.

15. Verify Stokes' theorem for the vector field $\mathbf{F}(x, y, z) = 2zi + 3xj + 5y\boldsymbol{k}$ taking surface σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy-plane.

16/12/17 (M)

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **6625**

Unique Paper Code : **32351303** **HC**

Name of the Paper : **Multivariate Calculus**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

Attempt any *five* questions from each Section.

All questions carry equal marks.

Section I

1. Let f be the function defined by :

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x^2 + y^6} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$$

Is f continuous at (0, 0) ? Explain.

2. Find the equation for each horizontal tangent plane to the surface :

$$z = 5 - x^2 - y^2 + 4y.$$

3. Let f and g be twice differentiable functions of one variable and let $u(x, t) = f(x + ct) + g(x - ct)$ for a constant c. Show that :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

4. Let f have continuous partial derivatives and suppose the maximal directional derivative of f at $P_0(1, 2)$ has magnitude 50 and is attained in the direction from P_0 towards Q(3, –4). Use this information to find $\nabla f(1, 2)$.

5. Find the absolute extrema of $f(x, y) = x^2 + xy + y^2$ on the closed bounded set S where S is the disk $x^2 + y^2 \leq 1$.

6. Find the point on the plane $2x + y + z = 1$ that is nearest to the origin.

Section II

7. Find the area of the region D by setting double integral, where D is bounded by the parabola $y = x^2 - 2$ and the line $y = x$.

8. Write an equivalent integral with the order of integration reversed and then compute the integral :

$$\int_0^4 \int_0^{4-x} xy \, dy dx.$$

9. Calculate the Jacobian of transformation from rectangular to polar coordinates and hence evaluate the integral :

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{9 - x^2 - y^2}} \, dx dy.$$

10. Find the volume V of the solid bounded above by the cylinder $y^2 + z = 4$ and below by $x^2 + 3y^2 = z$.

11. Evaluate the integral below, where D is the region bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$:

$$\iiint_D z \, dx \, dy \, dz.$$

16. State and prove Divergence theorem.

17. Verify that the vector field $\mathbf{F}(x, y) = (e^x \sin y - y)\boldsymbol{i} + (e^x \cos y - x - 2)\boldsymbol{j}$ is conservative using cross partial test. Use a line integral to find the area enclosed by the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

18. Let E be the solid unit cube with opposing corners at the origin and (1, 1, 1) with faces parallel to co-ordinate planes. Let S be the boundary surface of E oriented with the outward pointing normal. If $\mathbf{F}(x, y, z) = 2xy\boldsymbol{i} + 3ye^{z}\boldsymbol{j} + x \sin z\, \boldsymbol{k}$, find the integral $\iint \mathbf{F.n}\, ds$ over surface S using divergence theorem.