$$A = \begin{bmatrix} 5 & 8 & 0 & 1 \\ 0 & -4 & 7 & -5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(6½)

[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6622 **HC**

Unique Paper Code : 32351102

Name of the Paper : Algebra

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **Six** questions are compulsory.
3. Do any **two** parts from each question.

1. (a) Find all complex numbers z, such that $|z| = 1$ and

$$\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1.$$ (6)

(b) Find the fourth roots of unity and represent them in the complex plane. Show that they form a square inscribed in the unit circle. (6)

P.T.O.

(c) Solve the equation

$$z^6 + iz^3 + i - 1 = 0. \qquad (6)$$

2. (a) For $a, b \in Z$, define $a \sim b$ if and only if $3a + b$ is a multiple of 4.

   (i) Prove that $\sim$ defines an equivalence relation.

   (ii) Find the equivalence class of 0 and 2. (6)

(b) Let $\sim$ denote an equivalence relation on a set A and $a \in A$. Prove that for any $x \in A$, $x \sim a$ if and only if $\bar{x} = \bar{a}$, where $\bar{x}$ denotes the equivalence class of x. (6)

(c) Show that Z and 3Z have the same cardinality. (6)

3. (a) Using Euclidean algorithm find g.c.d [1004, −24) and express it as an integral linear combination of the given integers. (6)

(b) Find $(1017)^{12} \pmod 7$. (6)

(c) Using Principle of Mathematical Induction, prove that for every positive integer n, $n^3 + 2n$ is divisible by 3. (6)

(c) Let $T: \mathcal{R}^2 \to \mathcal{R}^2$ be defined as $T(x_1, x_2) = (x_2 - x_1, 2x_2 + x_1)$ be a linear transformation. Prove that T is invertible and find a rule for T^{-1}. (6½)

(a) Let

$$A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 0 & 7 \\ -3 & -5 & -3 \end{bmatrix} \text{ and } u = \begin{bmatrix} -7 \\ 3 \\ 2 \end{bmatrix}$$

Is u in Nul A ? Is u in Col A ? Justify each answer. (6½)

(b) (i) Suppose a 4 × 7 matrix A has three pivot columns. Is Col A = $\mathcal{R}^3$? What is the dimension of Nul A ? Explain your answer.

(ii) Consider the basis $B = \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$ for $\mathcal{R}^2$. If $[x]_B = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, find the vector x. (3½,3)

(c) For the matrix given below, find the characteristic equation and the eigen values with their multiplicities. Also, find a basis for the eigenspace corresponding to any one of the eigenvalues.

(c) Let $v_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$

Do the vectors v_1, v_2, v_3 span $\mathcal{R}^3$? Justify. Hence or otherwise express $v = \begin{bmatrix} 8 \\ -4 \\ 2 \end{bmatrix}$ as linear combination of v_1, v_2, v_3. (6½)

5. (a) Boron sulphide reacts violently with water to form boric acid and hydrogen sulphide gas. The unbalanced equation is

$$B_2S_3 + H_2O \rightarrow H_3BO_3 + H_2S$$

Balance the chemical equation using the vector equation approach. (6½)

(b) Let T: $\mathcal{R}^2 \rightarrow \mathcal{R}^2$ be a linear transformation such that T first rotates through $\frac{\pi}{2}$-radians in the anti-clockwise direction and then reflects through the line $x_1 = x_2$. Find the Standard matrix of T. (6½)

4. (a) Find the general solution to the linear system whose augmented matrix is

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 & -3 & 2 \\ 1 & 1 & 1 & 2 & -3 & 3 \\ 2 & 1 & 0 & 2 & -3 & 4 \\ 4 & 3 & 1 & 1 & -9 & 9 \end{bmatrix}$$

by row reducing the matrix to Echelon Form. Encircle the leading entries, list the basic variables and free variables. Write the general solution in Parametric Vector Form. (6½)

(b) Define Linearly Dependent Se

Let $v_1 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ -5 \\ 10 \end{bmatrix}$, $v_3 = \begin{bmatrix} -1 \\ 5 \\ h \end{bmatrix}$ for what value(s) of h, the set $\{v_1, v_2, v_3\}$ is

(i) Linearly Independent

(ii) Linearly Dependent. (6½)