25|5|17 Eve

This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2466**

Unique Paper Code : **32375201** **GC-4**

Name of the Paper : **Introductory Probability**

Name of the Course : **GE : Statistics for Honours**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Section A is compulsory.

Attempt any *five* questions, selecting at least *two* questions from each of the sections B and C.

Use of simple calculator is allowed.

Section A

1. Answer the following :

(*i*) If mean and variance of X is 10 and 50 respectively, then $E(X^2)$ = ... 1

(*ii*) Let F(1) = 0.3, F(2) = 0.5, F(3) = 0.8 and F(4) = 1.2. Can F(x) serve as distribution function of a r.v. with the range 1, 2, 3 and 4 ? State the reason. 1

(*iii*) A and B are events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(A') = \frac{2}{3}$. Find $P(A \cap B')$. 1

(*iv*) If Cov(X, Y) = 20, E(X) = 15 and E(Y) = 4, then E(XY) = ... 1

(*v*) Define convergence in probability. 1

(*vi*) Let X be a Normal variate with m.g.f. $M_X(t) = \exp(15t + 18t^2)$. Find the mean and variance of X. 2

(*vii*) If X is a random variable with m.g.f. $M_X(t) = \left(\frac{1}{3} + \frac{2}{3}e^t\right)^{10}$, then m.g.f. of Y = X − 3 is 2

(*viii*) If X and Y are independent r.v's with $\mu_X = 13$, $\mu_Y = 20$, $\sigma_X^2 = 49$ and $\sigma_Y^2 = 100$, then E(4X + 6Y) = ... and Var(4X + 6Y) = ... 2

(*ix*) If A and B are mutually exclusive events with P(A) = 0.37, and P(B) = 0.44, then P(A') = ... and $P(A \cup B)$ = ... 2

(*b*) The random variables X_i (i = 1,2,3,4,5) are independent and identically distributed with p.d.f.

$$f(x) = \frac{1}{\sqrt{18\pi}} \exp\left(-\frac{(x-1)^2}{18}\right).$$

Obtain the distribution of (*i*) $V = \frac{1}{5}\sum_{i=1}^{5} X_i$, and (*ii*) $W = 3X_1 - X_2 + 2X_3$. 8,4

8\. (*a*) Let X be a random variable having p.d.f.

$$f(x) = \frac{1}{B(2,3)} x(1-x)^2, \; 0 < x < 1.$$

Find (*i*) $P(X \leq 0.5)$ and (*ii*) $P(0.5 < X < 1)$.

(*b*) If X is uniformly distributed with $\mu'_1 = 1$ and $\mu_2 = 4/3$ then find P(X < 0).

(*c*) Let the p.d.f. of r.v. X be $f(x) = \theta \exp(-\theta x)$, $x \geq 0$. Identify the distribution of X. Derive its mean and variance. 4,4,4 (*b*) State Chebyshev's inequality. For $f(x) = 2^{-x}$, $x = 1, 2, 3, \ldots\ldots$, prove that Chebyshev's inequality gives $P(|X-2|<2) \geq 0.5$, while the actual probability is 7/8.

6,6

6. (*a*) Derive mean and variance of Poisson distribution with parameter λ.

(*b*) In a book of 520 pages, 390 typo-graphical errors occur. Assuming Poisson law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error.

(*c*) If X is a binomial variate with $n = 5$ such that $P(X = 1) = 0.4096$ and $P(X = 2) = 0.2048$. Find $P(X = 3)$.

4,4,4

7. (*a*) Derive the mean and variance of negative binomial distribution with *p.m.f.*

$$f(x) = \begin{cases} \binom{x+r-1}{r-1} p^r q^x, & \text{for } x = 0, 1, 2, \ldots \\ 0, & \text{elsewhere.} \end{cases}$$

(*x*) What is the smallest value of k in Chebyshev's inequality for which the probability that a random variable will take on a value between $\mu - k\sigma$ and $\mu + k\sigma$ is at least 0.95 ? 2

Section B

2. (*a*) State Bayes' theorem. At a hospital's emergency room, patients are classified and 20% of them are critical, 30% are serious and 50% are stable. Of the critical ones, 30% die; of the serious, 10% die; and of the stable, 1% die.

(*i*) Find the probability that a patient selected at random will die.

(*ii*) Given that a patient dies, what is the conditional probability that the patient was classified as critical ?

(*b*) The odds that a book on Statistics will be favourably reviewed by 3 independent critics are 3 to 2, 4 to 3 and 2 to 3 respectively. What is the probability that of the three reviews : 6,6

(*i*) All will be favourable,

(*ii*) Exactly one review will be favourable, and

(*iii*) At least one of the reviews will be favourable.

3. (*a*) Let X be a continuous random variable with p.d.f.

$$f(x) = \begin{cases} kx(1-x), & 0 < y \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

(*i*) Determine *k*.

(*ii*) Obtain distribution function F(x). Hence evaluate P(X > 0.6).

(*iii*) Compute $P(0.2 \le X \le 0.6)$

(*b*) Let X be a discrete random variable with distribution function :

$$F(x) = \begin{cases} 0 & \text{for } x < 2 \\ \frac{1}{4} & \text{for } 2 \le x < 4 \\ \frac{1}{2} & \text{for } 4 \le x < 6 \\ \frac{5}{6} & \text{for } 6 \le x < 10 \\ 1 & \text{for } x \ge 10 \end{cases}$$

(*i*) Obtain p.m.f. of X.

(*ii*) Compute $P(X \le 9)$ and $P(2.5 \le X \le 6.5)$.

(*iii*) Compute mean and variance of X. 6,6

4. (*a*) Find the moment generating function of the random variable X whose probability density function is given by

$$f(x) = \begin{cases} e^{-x}, & \text{for } x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

and use it to find the expression for μ'_r. Hence obtain mean and variance of X.

(*b*) Let the probability mass function of the random variable X be given by

$$f(x) = \frac{|x-2|}{7} \text{ for } x = -1, 0, 1, 3.$$

Find $E(X^2 - 5X + 3)$ and $Var(3X + 5)$. 7,5

Section C

5. (*a*) State De-Moivre's Laplace Central Limit theorm. Let $X_1, X_2 \ldots\ldots X_n$ be the sequence of independent random variables with distribution defined as :

$$P(X_k = 0) = 1 - k^{-2\alpha}, P(X_k = \pm k^{\alpha}) = \frac{1}{2}k^{-2\alpha},$$

where $\alpha < \frac{1}{2}$

Show that the central limit theorem holds.