4. (*a*) Using rank, find whether the non-homogeneous linear system $Ax = b$, where :

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -3 & 4 \\ 2 & -1 & 7 \end{pmatrix}, b = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix},$$

has a solution or not. 6½

(*b*) Suppose $L : \mathbf{R}^3 \to \mathbf{R}^2$ is a linear transformation with $L([1, -1, 0]) = [2, 1]$, $L([0, 1, -1]) = [-1, 3]$ and $L([0, 1, 0]) = [0, 1]$. Find $L([-1, 1, 2])$. Also give a formula for $L([x, y, z])$, for any $[x, y, z] \in \mathbf{R}^3$. 4+2½

(*c*) Let $L : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear operator given by $L([x, y]) = [2x - y, x - 3y]$. Find the matrix for L with respect to the basis $\{[4, -1], [-7, 2]\}$ using the method of similarity. 6½

5. (*a*) Consider the linear transformation $L : P_2 \to \mathbf{R}$ defined by :

$$L\left(p(x)\right) = \int_0^1 p(x)\, dx,$$

where P_2 is the vector space of polynomials of degree 2 or less. Show that L is onto but not one-to-one. 6

25/5/17 Eve

This question paper contains **4+2** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2447**

Unique Paper Code : **32355202** **GC-4**

Name of the Paper : **Linear Algebra**

Name of the Course : **Generic Elective : Mathematics for Honours**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

1. (*a*) If x and y are vectors in $\mathbf{R}^n$, then prove that :

(*i*) $\| x + y \|^2 = \| x \|^2 + \| y \|^2$ if and only if $x \cdot y = 0$, and

(*ii*) $x \cdot y = \frac{1}{4}\left(\| x + y \|^2 - \| x - y \|^2\right)$. 3+3

(*b*) Let x and y be non-zero vectors in $\mathbf{R}^n$, then prove that :

$$\| x + y \| = \| x \| + \| y \|$$

if and only if $y = cx$, for some $c > 0$. 6

(*c*) Using Gauss-Jordan method, find the complete solution set for the following system of homogeneous linear equations : 6

$$4x_1 - 8x_2 - 2x_3 = 0$$
$$3x_1 - 5x_2 - 2x_3 = 0$$
$$2x_1 - 8x_2 + x_3 = 0.$$

2. (*a*) Find the reduced row echelon form matrix B of the following matrix : 4+2½

$$A = \begin{pmatrix} 4 & 0 & -20 \\ -2 & 0 & 11 \\ 3 & 1 & -15 \end{pmatrix}$$

and then give a sequence of row operations that converts B back to A.

(*b*) Find the characteristic polynomial and eigenvalues of the matrix : 4+2½

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 2 & -4 \\ 3 & -4 & 7 \end{pmatrix}.$$

Is A diagonalizable ? Justify.

(*c*) Let V be the set $\mathbf{R}^2$ with operations addition and scalar multiplication for x, y, w, z and a in $\mathbf{R}$ defined by :

$$[x, y] \oplus [w, z] = [x + w - 2, y + z + 3], \text{ and}$$
$$a \odot [x, y] = [ax - 2a + 2, ay + 3a - 3].$$

Prove that V is a vector space over $\mathbf{R}$. Find the zero vector in V and the additive inverse of each vector in V. 4+2½

3. (*a*) Prove that the set S = {[3, 1, –1], [5, 2, –2], [2, 2, –1]} is linearly independent in $\mathbf{R}^3$. Examine whether S forms a basis for $\mathbf{R}^3$? 4+2

(*b*) Find a basis and the dimension for the subspace W of $\mathbf{R}^3$ defined by : 6

$$W = \{[x, y, z] \in \mathbf{R}^3 : 2x - 3y + z = 0\}.$$

(*c*) Let S = {[1, 2], [0, 1]} and T = {[1, 1], [2, 3]} be two ordered bases for $\mathbf{R}^2$. Let v = [1, 5]. Find the coordinate vector $[v]_S$ and hence find $[v]_T$ using the transition matrix $Q_{T \leftarrow S}$ from S-basis to T-basis. 3+3 where P_2 is the vector space of polynomials of degree 2 or less. Find a basis for ker(L) and a basis for range(L), and also verify the dimension theorem. 4+2½

(*c*) For the subspace $W = \{[x, y, z] \in \mathbf{R}^3 : 3x - y + 4z = 0\}$ of $\mathbf{R}^3$, find the orthogonal complement $W^{\perp}$ and veirfy that $\dim(W) + \dim(W^{\perp}) = \dim(\mathbf{R}^3)$. 4+2½ (*b*) Let W be the subspace of $\mathbf{R}^3$ whose vectors lie in the plane $2x + y + z = 0$. Find the minimum distance from the point P(–6, 10, 5) to W. 6

(*c*) Find a least squares solution for the linear system $Ax = b$, where : 6

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 1 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 5 \\ 4 \end{pmatrix}.$$

6. (*a*) Use the similarity method to show that a rotation about the point (1, –1) through an angle $\theta = 90°$, followed by a reflection about the line $x = 1$ is represented by

the matrix $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix}$. 6½

(*b*) Let $L : P_2 \to \mathbf{R}^2$ be the linear transformation given by :

$$L(p(x)) = [p(1), p'(1)],$$