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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7336**

Unique Paper Code : **32355101** **HC**

Name of the Paper : **Calculus**

Name of the Course : **Generic Elective for Honours : Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do any *five* questions from each of the *three* Sections.

Each question is of *five* marks.

Section I

1. Use $\varepsilon - \delta$ definition to show that :

$$\lim_{x \to 4} (9 - x) = 5.$$

2. Find the asymptotes for the curve :

$$f(x) = \frac{x^2 - 3}{2x - 4}.$$

3. Find the Linearization L(x) of $f(x)$ at $x = a$ where :

$$f(x) = x + \frac{1}{x} \text{ at } a = 1.$$

4. For $f(x) = (x - 2)^3 + 1$

(*i*) Find the intervals on which f is increasing and the intervals on which f is decreasing.

(*ii*) Find where the graph of f is concave up and where it is concave down.

5. Use L'Hôpital's rule to find :

$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right).$$

20. If

$$f(x, y) = x^2 - y^2, \quad g(x, y) = 3xy + y^2 x,$$

show that :

(*i*) $\nabla(fg) = f\nabla g + g\nabla f$

(*ii*) $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}.$

21. If $w = x \sin y + y \sin x + xy$, show that $w_{xy} = w_{yx}$.

16. Find the derivative of the function f at p_0 in the direction of $\vec{A}$ where $f(x, y, z) = 3e^x \cos yz$, $p_0(0, 0, 0)$, $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$.

17. Find parametric equations for the line tangent to the curve of intersection of the surfaces at the given point.

Surfaces : $x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0$, $x^2 + y^2 + z^2 = 11$

Point : (1, 1, 3).

18. Find equations for the :

(a) Tangent plane and

(b) Normal line at the point p_0 on the given surface :

$$f(x, y, z) = x^2 + y^2 + z - 9 = 0 \text{ at the point } p_0(1, 2, 4).$$

19. Find the absolute maximum and minimum value of :

$$f(x, y) = 2 + 2x + 2y + x^2 - y^2$$

on the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$.

6. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

7. Find the length of the curve :

$$x = t^3,\ y = \frac{3t^2}{2},\ 0 \le t \le \sqrt{3}.$$

Section II

8. State Limit comparison test. Using the limit comparison test, show that :

$$\int_1^\infty \frac{3dx}{e^x + 5} \text{ converges.}$$

9. Identify the symmetries of the curve and then sketch the graph of :

$$r^2 = \cos\theta.$$

10. If $\vec{r}(t)$ is the position vector of a particle in space at time t, find the time in the given interval when the velocity and acceleration are orthogonal, where :

$$\vec{r}(t) = (t - \sin t)\hat{i} + (1 - \cos t)\hat{j}, \quad 0 \le t \le 2\pi.$$

11. Find the area of the surface swept out by revolving the circle $x^2 + y^2 = 1$ about the x-axis.

12. Write the acceleration vector $\vec{a}$ in the form of $\vec{a} = a_T \hat{T} + a_N \hat{N}$ at the given value of t without finding $\hat{T}$ and $\hat{N}$ for the position vector given by :

$$\vec{r}(t) = \left(e^t \cos t\right)\hat{i} + \left(e^t \sin t\right)\hat{j} + \sqrt{2}\, e^t \hat{k}, \quad t = 0.$$

13. If

$$f(x, y) = \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}},$$

then find :

(i) $\lim_{(x, y) \to (0, 0)} f(x, y)$

(ii) Domain of $f(x, y)$.

14. Show that the function :

$$f(x, y) = \frac{2x^2 y}{x^4 + y^2}$$

is not continuous at (0, 0).

Section III

15. If $w = (x + y + z)^2$; $x = r - s$, $y = \cos(r + s)$, $z = \sin(r + s)$, find $\frac{\partial w}{\partial r}$ when $r = 1$, $s = -1$. Also draw the tree diagram.