[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **1812** **GC-4**

Unique Paper Code : 32341202

Name of the Course : **B.Sc.(Hons.) Computer Science**

Name of the Paper : Discrete Structures

Semester : II

Time : 3 Hours *Maximum Marks : 75*

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **Section A** (Question **1**) is compulsory and Carries **35** marks (7×5=35).

(c) Attempt any **four** questions from **Section B** (Question **2-7**)

(d) Parts of a question must be answered together.

(e) Symbols have their usual meanings.

P.T.O.

SECTION - A

1. (a) Among 100 students, 32 study mathematics, 20 study physics, 45 study biology, 15 study mathematics and biology, 7 study mathematics and physics, 10 study physics and biology and 30 do not study any of the three subjects. 5

(i) Find the number of students studying all three subjects.

(ii) Find the number of students studying exactly one of the 3 subjects.

(b) Let A= { 1,2,3,4,5,6,7} and R={(x,y) : x-y is divisible by 3}. show that R is an equivalence relation. 5

(b) (i) Write the Contrapositive, Converse and Inverse of the following statement :

"The home team wins whenever it is raining" 5

(ii) Show that

$$\neg(p \leftrightarrow Q) \equiv (P \lor Q) \land \neg(P \land Q)$$

6. (a) Solve the following recurrence relation :

$a_r - 7a_{r-1} + 10a_{r-2} = 3^r$, given that $a_0 = 0$ and $a_1 = 1$.

5

(b) Suppose that the number of bacteria in a colony triples every hour. 5

   (i) Set-up a recurrence relation for the number of bacteria after n hours have elapsed.

   (ii) If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours ?

7. (a) Show the validity of the following argument: If Ram gets the job and works hard, then he will be promoted. If Ram gets promotion, then he will be happy. He will not be happy. Therefore, either he will not get the job or he will not work hard.

5

(c) Define Hamilton path and Hamilton circuit. Classify the following graph as a Hamilton path and/or a Hamilton circuit. 5
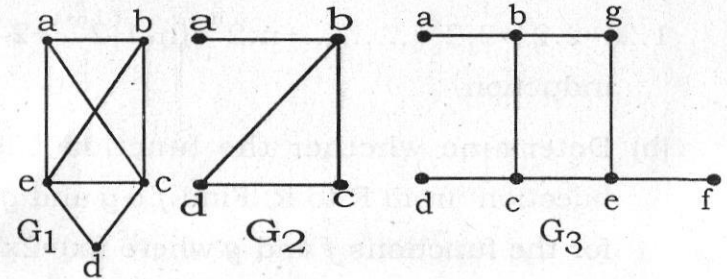
(d) Define Bipartite Graph and Complete Bipartite Graph. Check whether the graph C_6 is Bipartite or not ? Justify your answer.

5

(e) Determine the numeric function for the following generating function : 5

$A(Z) = 1/(5 - 6Z - Z^2)$

(f) Show that the following system is inconsistent : 5

$P \rightarrow Q, P \rightarrow R, Q \rightarrow \neg R, P$

(g) Use Master method to find the asymptotic bounds for the following recurrence 5

$T(n) = 2T(n/2) + n^2$

SECTION - B

2. (a) Show that :

$1.2^1+2.2^2+3.2^3+\ldots\ldots+n.2^n=(n-1)2^{n+1}+2$ by induction. 5

(b) Determine whether the function f is a bijection from R to R. Find $f\, o\, g$ and $g\, o\, f$ for the functions f and g where $f(x)=2x^2+3$ and $g(x)=x+1$. Is $f\, o\, g = g\, o\, f$? 5

3. (a) In how many ways can four students be selected out of twelve students, if : 5

(i) Two particular students are not included at all ?

(ii) Two particular students are included.

(b) Let A= {1,2,3,4,6,8,9,12,18,24} and let R be a partial ordered relation on A defined by xRy if and only if "x divides y". 5

(i) Draw the Hasse Diagram of R.

(ii) Find the Maximal and Minimal elements in A.

4

4. (a) Define planar graph. Is $K_{3,3}$ planar. Justify your Answer. For any connected planar graph, show that v-e+r=2, where v,e and r are the number of vertices, edges and regions of the graph respectively. 5

(b) What do you mean by graph invariant ? Determine whether the graphs G1 and G2 are isomorphic. 5

G1   G2
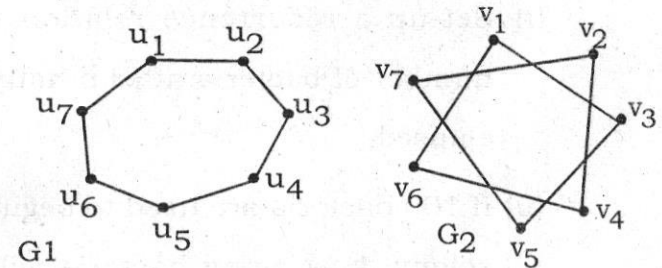
5. (a) (i) What is the chromatic number of C_n where $n \geq 3$ and $k_{m,n}$?

(ii) Prove that a connected graph is a tree if and only if the number of vertices in the graph is one more than the number of edges. 5

(b) Let $f(n)=n^2+4n$ and $g(n)=n^2$, $n \geq 0$. Show that $f(n)=O(g(n))$ 5

5   P.T.O.