6. (*a*) If X and Y are two independent random variables with probability density functions

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, -\infty < x < \infty \quad \text{and}$$

$$f(y) = \frac{1}{\sqrt{8\pi}} e^{-\frac{(y-5)^2}{8}}, -\infty < y < \infty$$

respectively, find the variance of random variable T = 2X + Y.

(*b*) The mean yield for one-acre plot is 662 kilos with s.d. 32 kilos. Assuming normal distribution, how many one-acre plots in a batch of 1000 plots, would you expect to have yield (*i*) over 700 kilos, and (*ii*) below 650 kilos ?

Given $P(0 \le Z \le 1.19) = 0.3830$, $P(0 \le Z \le 0.38) = 0.1480$, where Z is a standard normal variate. 6,6

27/5/17 Eve.

This question paper contains **4+1** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2017**

Unique Paper Code : **62371201** **GC-4**

Name of the Paper : **Statistical Methodology**

Name of the Course : **B.A. (Program) Statistics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Six* questions in all including Q. No. **1**

Which **1** is compulsory.

Simple calculator can be used.

1. (*a*) Find the distribution function for the following probability distribution :

X = *x*	**P(X = *x*)**
0	1/16
1	4/16
2	6/16
3	4/16
4	1/16

(*b*) m.g.f. of a random variable X is $M_x(t) = \exp\{3(e^t - 1)\}$. Using the uniqueness property of m.g.f.'s identify the distribution. Find its parameters and coefficient of skewness.

(*c*) If X and Y are independent standard normal variates, then find the distribution of X – 2Y.

(*d*) Find the r^{th} moment about origin for beta distribution of second kind.

(*e*) Determine the binomial distribution for which mean is 4 and variance is 3. Also obtain its mode. 3,3,3,3,3

2. (*a*) Prove that geometric mean G of the following distribution

$$dF = 6(2-x)(x-1)dx, 1 \le x \le 2$$

is given by 6 log (16G) = 19.

(*b*) Find m.g.f. of the standard binomial variate $\frac{x-np}{\sqrt{npq}}$ and show that it tends to $\exp(t^2/2)$ as n tends to infinity. 6,6

3. (*a*) In four tosses of coin, let X be the number of heads. Tabulate 16 possible outcomes with the corresponding values of X. Derive the probability distribution of X and hence, calculate the expected value of X.

(*b*) Show that in Poisson distribution with unit mean, mean deviation about mean is $(2/e)$ times the standard deviation. 6,6

4. (*a*) Define negative binomial distribution. Compute its m.g.f. and hence compute its mean and variance.

(*b*) State and prove De-Moivre's theorem. 6,6

5. (*a*) For geometric distribution $p(x) = 2^{-x}; x = 1,2,3....,$ prove that Chebychev's inequality gives $P\{|x-2| \le 2\} > \frac{1}{2}$, while the actual probability is 15/16.

(*b*) Show that hypergeometric distribution with parameters (N, M, n) tends to binomial distribution with parameters (n, p) as $N \to \infty$ and $\frac{M}{N} \to p$ 6,6

7. (*a*) Obtain m.g.f. of gamma distribution and hence compute β_1 and β_2.

(*b*) Compute mean deviation about mean for exponential distribution. 6,6

8. (*a*) Obtain harmonic mean for beta distribution of first kind.

(*b*) If X ~ Bin (n, p), find :

$$E\left[\frac{x}{n}-p\right]^2 \text{ and Cov } \left(\frac{x}{n}, \frac{n-x}{n}\right).$$ 6,6