

5/12/16 (Even)  
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[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 198

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Your Roll No.....

Unique Paper Code : 235351

Name of the Paper : Integration and Differential Equations

Name of the Course : B.A. (Prog.) - Mathematics

Semester : III

Duration : 3 Hours

Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two parts from each question.

1. (a) Find the area enclosed by the curves  $y^2 = 4x$  and  $y = 2x - 4$ . (6)

(b) Obtain a reduction formula for  $\int \sec^n x dx$ ,  $n$  being a positive integer and hence evaluate  $\int \sec^6 x dx$ . (6)

(c) Evaluate :

$$\int \frac{1}{3\sin x - 4\cos x} dx. \quad (6)$$

2. (a) Show that

$$\int_0^{\frac{\pi}{2}} \cos^m x \cos nx dx = \frac{m}{m+n} \int_0^{\frac{\pi}{2}} \cos^{m-1} x \cos(n-1)x dx.$$

Further show that

$$\int_0^{\frac{\pi}{2}} \cos^n x \cos nx dx = \frac{\pi}{2^{n+1}},$$

where  $m$  and  $n$  being positive integers.

(6½)

P.T.O.

- (b) Find the exact arc length of the curve

$$24xy = y^4 + 48 \text{ from } y = 2 \text{ to } y = 4. \quad (6\frac{1}{2})$$

- (c) Let  $V_x$  and  $V_y$  be the volume of the solids that result when the region enclosed by

$$y = \frac{1}{x}, y = 0, x = \frac{1}{2} \text{ and } x = b \text{ (} b > 2 \text{)}$$

is revolved about  $x$  - axis and  $y$  - axis, respectively. Is there any value of  $b$  for which  $V_x = V_y$  ? (6½)

3. (a) Evaluate :

(i)  $\int_0^{\frac{\pi}{2}} \log(\tan x + \cot x) dx$

(ii)  $\int \frac{2x+3}{\sqrt{4x^2+5x+6}} dx \quad (3+3)$

- (b) Solve :

$$y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0. \quad (6)$$

- (c) Given that  $y = x + 1$  is a solution of

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1) \frac{dy}{dx} + 3y = 0.$$

Find a linearly independent solution by reducing the order. (6)

4. (a) Solve :

$$x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = 4x - 6. \quad (6)$$

- (b) Using the concept of Wronskian, show that  $e^x \sin x$  and  $e^x \cos x$  are linearly independent solution of

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0.$$

Find the solution  $y(x)$  satisfying the conditions  $y(0) = 2$  and  $y'(0) = -3$ .

(6)

- (c) Assume that the population of a certain city increases at a rate proportional to the number of inhabitants at any time. If the population doubles in 40 years, in how many years will it triple ? (6)

5. (a) Using method of variation of parameters, find the general solution of

$$\frac{d^2 y}{dx^2} + y = \tan x. \quad (6\frac{1}{2})$$

- (b) Solve :

$$a^2 y^2 z^2 dx + b^2 x^2 z^2 dy + c^2 x^2 y^2 dz = 0. \quad (6\frac{1}{2})$$

- (c) Solve the system of equations :

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0$$

$$\frac{dy}{dt} + 5x + 3y = 0 \quad (6\frac{1}{2})$$

6. (a) (i) Classify the following partial differential equation into elliptic, parabolic or hyperbolic form :

$$r + 2s + t = 0$$

$$\text{where } r = \frac{\partial^2 z}{\partial x^2}, s = \frac{\partial^2 z}{\partial x \partial y}, t = \frac{\partial^2 z}{\partial y^2}. \quad (3)$$

- (ii) Eliminate the arbitrary function  $f$  to form the partial differential equation from the following equation

$$z = f\left(\frac{xy}{z}\right). \quad (3\frac{1}{2})$$

- (b) Find the general integral of the linear partial differential equation

$$z(xp - yq) = y^2 - x^2. \quad (6\frac{1}{2})$$

- (c) Find the complete integral of the partial differential equation

$$p + q = pq. \quad (6\frac{1}{2})$$