

4. (a) Find the asymptotes of the curve : 6½

$$x^3 - 4x^2y + 5xy^2 - 2y^3 + 3x^2 - 4xy + 2y^2 - 3x + 2y - 1 = 0.$$

- (b) Find the equation of the tangent to the curve : 6½

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$$

at  $(-1, -2)$ , and show that it is a cusp.

- (c) Trace the curve : 6½

$$x^3 + y^3 = 3axy, a > 0.$$

5. (a) State Lagrange's mean value theorem. Can we drop some condition of Lagrange's mean value theorem? Justify your answer. 6

- (b) Let  $f(x) = \tan x$  for all  $x$  in  $\mathbf{R}$ . Using Lagrange's mean value theorem, for the function  $f$ , show that : 6

$$|\tan^{-1} x - \tan^{-1} y| < |x - y| \quad \forall x, y \in \mathbf{R}.$$

This question paper contains 4+1 printed pages]

20/12/17 (E)

Roll No.

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S. No. of Question Paper : 7948

Unique Paper Code : 62351101

HC

Name of the Paper : Calculus

Name of the Course : B.A. (Prog.) Mathematics

Semester : I

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.

1. (a) Discuss the existence of the limit of the function : 6

$$f(x) = \frac{1}{e^{x^2} - 1}$$

at  $x = 0$ .

P.T.O.

- (b) Discuss the continuity of : 6

$$f(x) = |x - 1| + |x - 2|$$

at  $x = 1$  and  $x = 2$ . Also state the kind of discontinuity, if any.

- (c) Examine the following function for differentiability at  $x = 0$  : 6

$$f(x) = \begin{cases} x \frac{e^x - 1}{e^x + 1} ; & x \neq 0 \\ 0 & ; \quad x = 0 \end{cases}$$

2. (a) Find the  $n$ th derivative of  $\cos(x + 5)$ . 6

- (b) If 6

$$y = \left[ x + \sqrt{1 + x^2} \right]^m,$$

prove that :

$$(1 + x^2) y_{n+2} + (2n + 1)x y_{n+1} + (n^2 - m^2) y_n = 0.$$

- (c) If 6

$$u = \log \left( \frac{x^2 + y^2}{x + y} \right),$$

then using Euler's theorem, prove that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1.$$

3. (a) If the tangent to the curve : 6½

$$\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$$

cuts off intercepts  $p$  and  $q$  from the axis of  $x$  and  $y$  respectively, show that :

$$\frac{p}{a} + \frac{q}{b} = 1.$$

- (b) Find the equation of the tangent to the curve  $y^2 = 4x$  which makes an angle  $45^\circ$  with the  $x$ -axis. 6½

- (c) Show that radius of curvature is  $4a \cos \frac{\theta}{2}$  for the cycloid : 6½

$$x = a(\theta + \sin \theta), \quad y = a(1 - \cos \theta).$$

- (c) Let  $f$  be a function defined by : 6

$$f(x) = x^3 - 6x^2 + 9x + 1 \quad \forall x \in \mathbf{R}.$$

Find the interval in which the function  $f$  is increasing or decreasing.

6. (a) Find the maximum and minimum values of the function : 6½

$$f(x) = 2x^3 - 15x^2 + 36x + 10 \quad \forall x \in \mathbf{R}.$$

- (b) Define extremum of a function. Give an example of a function with no extremum. Justify your answer. 6½
- (c) Evaluate : 6½

$$\lim_{x \rightarrow 0^+} (\cot x)^{\sin x}.$$