(b) (i) Determine whether the points P_1 (–6, 4, 8), $P_2(9, – 2, 0)$ and P_3 (1, –5, 3) lie on the same line.
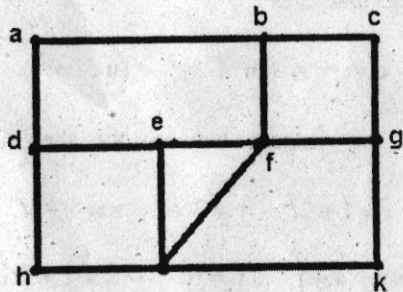
(ii) Where does the line

$$x = 2 - t, \ y = 3t, \ z = 1 + 2t$$

intersect the plane $2x - 7y + 3z = 6$.   3+3½

(c) Find the equation of the plane through the points $P_1(-2, 1, 4)$, $P_2(1, 0, 3)$ that is perpendicular to the plane $4x - y + 3z = 2$.   6½

6 (a) Find a maximum independent set of vertices for the following graph. What is the minimum number of independent set needed to cover all the vertices ? 6½

All questions are compulsory.

Attempt any *two* parts from each question.

1. (a) Identify and sketch the curve :

$$(x + 2)^2 = - (y + 2)$$

and also label the focus, vertex and directrix.   6

(b) Sketch the ellipse :

$$9(x - 3)^2 + 25(y + 1)^2 = 225$$

also label foci, vertices and ends of major and minor axes.   6

(c) Describe the graph of the equation :

$$x^2 - 4y^2 + 2x + 8y - 7 = 0.$$ 6

2. (a) Find the equation of the parabola that has its vertex at $(1, 1)$ and directrix $y = -2$. Also state the reflection property of parabola. 　　6

(b) Find an equation for the ellipse with length of major axis 10 and with vertices $(3, 2)$ and $(3, -4)$ and also sketch it. 　　6

(c) Find and sketch the curve of the hyperbola whose asymptotes are $y = 2x + 1$ and $y = -2x + 3$ and the hyperbola passes through the origin. 　　6

3. (a) Consider the equation $x^2 - 10\sqrt{3}xy + 11y^2 + 64 = 0$. Rotate the coordinate axes to remove the xy term and then identify the type of the conic represented by the above equation. 　　6

(b) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 60°$.

(i) Find the $x'y'$-coordinate of the point whose xy-coordinates are $(2,6)$.

(ii) Find an equation of the curve $\sqrt{3}xy + y^2 = 6$ in $x'y'$-coordinates. 　　6

(c) Find the equation of the sphere with center at $(2,-1,-3)$ and is tangent to the zx-plane. 　　6

4. (a) (i) Find a vector $\mathbf{v}$ having opposite direction as the vector from the point P $(1, 0, -6)$ to Q $(-3, 1, 1)$ with $\| \mathbf{v} \| = 5$.

(ii) Sketch the surface $z^2 + y^2 = 4$ in 3-space. 　　3+3½

(b) (i) Using vector, find the area of triangle with vertices A$(2, 2, 0)$, B$(-1, 0, 2)$ and C$(0, 4, 3)$.

(ii) Let $\mathbf{u} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$ and $\mathbf{w} = 2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$. Find the volume of the parallelopiped with adjacent edges $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$. 　　3+3½

(c) Prove that

$$\mathbf{u}.\mathbf{v} = \tfrac{1}{4} (\|\mathbf{u} + \mathbf{v}\|^2 - \|\mathbf{u} - \mathbf{v}\|^2).$$ 　　6½

5. (a) Find the distance between the skew lines : 　　6½

$$L_1 : x = 1 + 7t \quad y = 3 + t \quad z = 5 - 3t, \quad -\infty < t < \infty.$$

$$L_2 : x = 4 - t \quad y = 6 \quad z = 7 + 2t, \quad -\infty < t < \infty$$

P.T.O.

(b) (i) Find a matching or explain why none exists for the following graph :
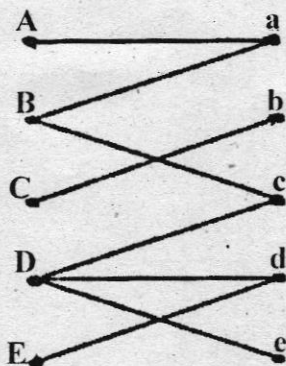
(ii) Given three pitchers: 8, 5 and 3 liters capacity. Only 8 liter pitcher is full. Make at least one of them contain exactly 4 liter of water with the minimum number of water transfers. 3+3½

(c) Defing Latin square. Construct a Latin square of order 5 on $\{e, e^2, e^3, e^4, e^5\}$. 6½