(a, b), (a, e) and (c, d), (d, h) ? Either produce other

or give an argument why no other exist. 6½,6½,6½
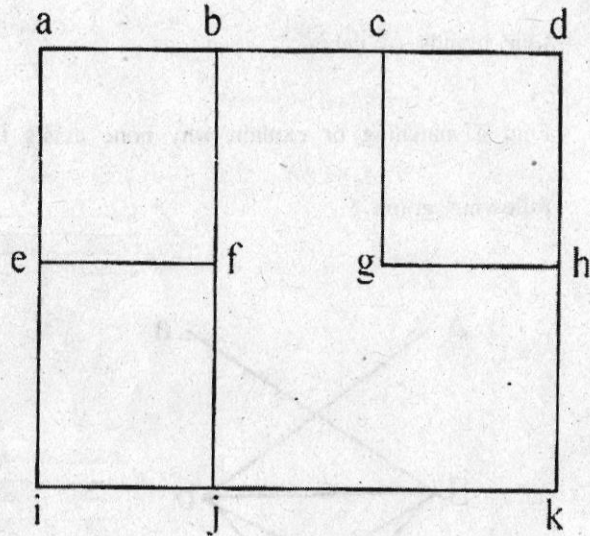
This question paper contains **8** printed pages]

27/5/17   Eve.

Roll No. ☐☐☐☐☐☐☐☐☐☐☐

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (a) Describe the graph of the equation :

$$y^2 - 8x - 6y - 23 = 0.$$

(b) Sketch the ellipse :

$$4x^2 + y^2 + 8x - 10y = -13$$

and label the foci, the vertices, and the ends of the minor-axis.

P.T.O.

(c) Find the centre vertices, foci and asymptotes of the hyperbola whose equation is :

$$4x^2 - 9y^2 + 16x + 54y - 29 = 0$$

and sketch its graph.   6,6,6

2. (a) Find an equation for the parabola whose axis is $y = 0$ and it passes through the points (3, 2) and (2, −3).

(b) Find on equation of the ellipse whose foci are (1, 2) and (1, 4) and minor-axis is of the length 2.

(c) Find an equation for a hyperbola whose foci are (0, ±5) and asymptotes are $y = ±2x$.   6,6,6
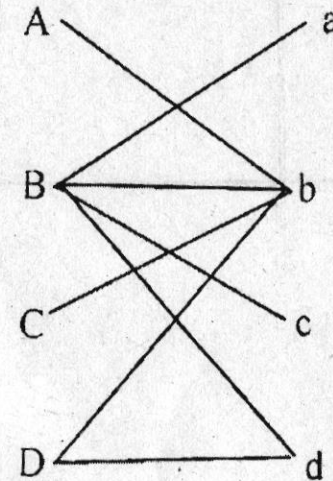
3. (a) Rotate the coordinate axes to remove the xy-term of the curve

$$31x^2 + 10\sqrt{3}\,xy + 21y^2 - 144 = 0$$

and then name the conic.

6. (a) A supermarket wishes to test the effect of putting cereal on four shelves at different heights. Show show to design such an experiment lasting four weeks and using four brands of cereal.

(b) Find a matching or explain why none exists for the following graph :

(c) What are the other sets of 2 edges whose removal disconnects the graph in the following figure besides

(c) (i) Show that the line

$$x = -1 + t,$$

$$y = 3 + 2t,$$

$$z = -t,$$

and the plane

$$2x - 2y - 2z + 3 = 0$$

are parallel and find the distance between

them.

(ii) Find the equation of the plane through the

points,

$$P_1(-2, 1, 4), P_2(1, 0, 3)$$

that is perpendicular to the plane :

$$4x - y + 3z = 2.　　　　　7,7,7$$

(b) Let an $x'y'$-coordinate system be obtained by rotating

an xy-coordinate system through an angle of 45°. Find

an equation of the curve

$$3x'^2 + y'^2 = 6$$

in xy-coordinate system.

(c) (i) Find the angle between a diagonal of a cube and

one of its edges.

(ii) Find k so that the vector from the point

A(1, −1, 3) to the point B(3, 0, 5) is orthogonal

to the vector from A to the point P(k, k, k).　6,6,6

4. (a) Find an equation of the sphere that is inscribed in the

cube that is centred at the point (−2, 1, 3) and has

sides of length 1 that are parallel to the coordinate

planes.

(b) (i) Prove that :

$$\left\| \vec{u} + \vec{v} \right\|^2 + \left\| \vec{u} - \vec{v} \right\|^2 = 2 \left\| \vec{u} \right\|^2 + 2 \left\| \vec{v} \right\|^2$$

where $\vec{u}$ and $\vec{v}$ are any two vectors.

(ii) Find the vector component of $\vec{a}$ and $\vec{b}$ and

the vector component of $\vec{a}$ orthogonal to $\vec{b}$

where

$$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k},$$

$$\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}.$$

(c) (i) Find the volume of the tetrahedron with

vertices

P(−1, 2, 0), Q(2, 1, −3), R(1, 0, 1), S(3, −2, 3).

(ii) Find two unit vectors that are normal to the plane

determined by the points

A(0, −2, 1), B(1, −1, −2) and C(−1, 1, 0) 6,6,6

5. (a) Let L_1 and L_2 be the lines whose parametric equations

are :

$L_1 : x = 4t, \qquad y = 1 - 2t, \qquad z = 2 + 2t$

$L_2 : x = 1 + t, \qquad y = 1 - t, \qquad z = -1 + 4t.$

Find parametric equations for the line that is perpendicular to L_1 and L_2 and passes through their point of intersection.

(b) (i) Find the parametric equations of the line that

passes through (−1, 2, 4) and is parallel to

$$3\hat{i} - 4\hat{j} + \hat{k}.$$

Also find the intersection of the line with

xy-plane.

(ii) Find an equation of the plane through the point

(−1, 4, 2) that contains the line of intersection

of the planes :

$4x - y + z - 2 = 0$ and

$2x + y - 2z - 3 = 0.$