9/12/17 (E) ①

[This question paper contains 7 printed pages]

Your Roll No.	:	
Sl. No. of Q. Paper	: **7986**	**HC**
Unique Paper Code	: 62354343	
Name of the Course	: **B.A.(Programme) Mathematics**	
Name of the Paper	: Analytical Geometry and Applied Algebra	
Semester	: III	

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) **All** questions are compulsory.

(iii) Attempt any **two** parts from each question.

1. (a) Identify and sketch the curve :

$x^2 - 4x + 2y = 1$

and also label the focus, vertex and directrix. 6

P.T.O.

(b) Describe the curve $x^2 + 9y^2 + 2x - 18y + 1 = 0$ 6

(c) Sketch the hyperbola :

$(y + 3)^2 - 9(x + 2)^2 = 36$

Also label the vertices, foci and asymptotes. 6

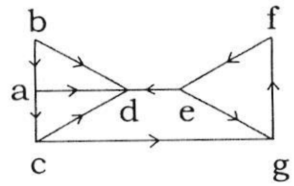
2. (a) Find and sketch an equation for the parabola with focus at (–1,4) and directrix at x = 5. 6

(b) Find the equation of the ellipse whose foci are (± 1,2) and sum of distance from each point on ellipse to foci is 6 units. Also sketch it. 6

(c) Find and sketch the curve of the hyperbola with vertices (0, ± 8) and asymptotes $y = \pm\frac{4}{3}x$. Also state the reflection property of hyperbolas. 6

3. (a) Rotate the coordinate axes through an angle θ to produce an equation of the curve $2x^2 + \sqrt{3}xy + y^2 - 10 = 0$, that has no product term. Find θ and the new equation, identify the curve and draw its rough graph. 6

(ii) Given the influence model. Find the sets of minimum number of vertices which can influence every other vertex in the graph.

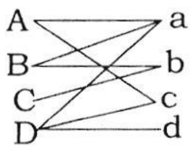


$3 + 3\frac{1}{2}$

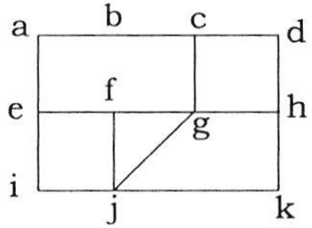
6. (a) Define Latin Square. Prove that every group is a Latin Square. What about the converse ? Justify.

$6\frac{1}{2}$

(b) (i) Find a matching or explain why none exists for the following graph.



(ii) Find all sets of three vertices that have all 11 vertices under surveillance.



$3 + 3\frac{1}{2}$

(c) (i) Given three pitchers of 12L, 8L, 5L (L=liter) only 12L pitcher is full. Find a minimum sequence of pouring to have 1L in either 8L or 5L.

(b) Let an x'y' – coordinate system be obtained by rotating an xy – coordinate system through an angle $\theta = 30^0$. 6

(i) Find the x'y' – coordinate of the point whose xy – coordinates are $(1, -\sqrt{3})$.

(ii) Find an equation of the curve $2x^2 + 2\sqrt{3}xy = 3$ in x'y' – coordinates.

(c) Find the equation of the sphere with center at (2, –1,–3) and is tangent to the yz – plane.

6

4. (a) (i) Find u and v if 5u + 2v = 6i – 5j + 4k and 3u – 4v = i + 2j + 9k.

(ii) Sketch the surface y = sinx in 3-space.

$3 + 3\frac{1}{2}$

(b) (i) Find k so that the vector from the point A(-1, 1, - 3) to the point B (3,0, -5) is orthogonal to the vector from A to point P (k, 2k, 3k).

(ii) Find the direction cosines of $u = 3i - 2j + 6k$ and verify that they satisfy $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$3 + 3\frac{1}{2}$

(c) (i) Find two unit vectors that are orthogonal to both $u = -7i + 3j + k$ and $v = 2i + 4k$.

(ii) Use scalar triple product to find the volume of parallelopiped that has $u = <2, -6, 2>$, $v = <0, 4, -2>$ and $w = <2, 2, -4>$ as adjacent edges.

$3 + 3\frac{1}{2}$

5. (a) (i) Find the parametric equation of the line passing through (–1, 2, 4) that is parallel to $3i - 4j + k$.

(ii) Find the points where the line

$x = 1 + t,\ y = 3 - t,\ z = 2t$

intersect the cylinder $x^2 + y^2 = 16$

$3+3\frac{1}{2}$

(b) Let L_1 and L_2 be the lines whose parametric equations are

$L_1 : x = 1 + 2t,\ y = 2 - t,\ z = 4 - 2t$

$L_2 : x = 9 + t,\ y = 5 + 3t,\ z = -4 - t$

(i) Show that the lines L_1 and L_2 intersect at the point (7, –1, –2).

(ii) Find the acute angle between L_1 and L_2 at their point of intersection.

$6\frac{1}{2}$

(c) (i) Find the equation of the plane that passes through the points (–2, 1,1), (0,2,3) and (1,0,-1)

(ii) Determine whether the line $x = 4 + 2t,\ y = -t\ z = -1 - 4t$ and plane $3x + 2y + z - 7 = 0$

are parallel, perpendicular or neither.

$3+3\frac{1}{2}$