5. (*a*) State and prove Cauchy's *n*th root test for an infinite series. 6

(*b*) Test for convergence the following series :

(*i*) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+5}$

(*ii*) $\sum_{n=1}^{\infty} 2^{-n-(-1)^n}$. 6

(*c*) State (without proof) D'Alambert's ratio test for an infinite series. Test for convergence the series :

$$\frac{1}{5}+\frac{2!}{5^2}+\frac{3!}{5^3}+\frac{4!}{5^4}+\ldots\ldots\ldots\ldots.$$ 6

6. (*a*) Define an alternating series. State Leibnitz's test for an alternating series. Test for convergence the series :

$$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{6}+\ldots\ldots\ldots\ldots.$$ 6

(*b*) Test for convergence the following series :

(*i*) $\sum_{n=1}^{\infty} \frac{(-1)^n \sin n\alpha}{n^3}$, $\alpha$ being real

(*ii*) $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}$. 6

(*c*) Prove that every monotonically increasing function $f$ on $[a, b]$ is Riemann integrable on $[a, b]$. 6



24|5|17 (Evenings)

This question paper contains 4 printed pages]

**Roll No.** | | | | | | | | | | | |

S. No. of Question Paper : **3019**

Unique Paper Code : **62354443** **GC-4**

Name of the Paper : **Analysis**

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

This question paper has *six* questions in all.

Attempt any *two* parts from each question.

*All* questions are compulsory.

Marks are indicated against each part of the questions.

1. (*a*) Which of the following sets are bounded below, which are bounded above and which are neither bounded below nor bounded above.

(*i*) $\{-1, -2, -3, -4, \ldots\ldots\ldots, -n, \ldots\ldots.\}$

(*ii*) $\{-1, 2, -3, 4, \ldots\ldots\ldots, (-1)^n n, \ldots\ldots.\}$

(*iii*) $\{2, \frac{3}{2}, \frac{4}{3}, \ldots\ldots\ldots, \left(\frac{n+1}{n}\right), \ldots\ldots.\}$

(*iv*) $\{3, 3^2, 3^3, \ldots\ldots\ldots, 3^n, \ldots\ldots.\}$

(v) $\{1, \frac{1}{2}, \frac{1}{2^2}, \ldots\ldots, \frac{1}{2^n} \ldots\ldots\}$

(vi) $\{-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \ldots\ldots, \frac{(-1)^n}{2}, \ldots\ldots\}$ 6.5

(b) Define supremum and infimum of a non-empty bounded set. Suppose A and B are two non-empty subsets of **R** satisfying the property :

$$a \leq b, \forall a \in \text{A} \quad \text{and} \quad \forall b \in \text{B}.$$

Prove that :

$$\sup(\text{A}) \leq \inf(\text{B}).$$ 6.5

(c) State Bolzano Weirstrass theorem for sets. Show by an example that the conditions in this theorem cannot be relaxed. 6.5

2. (a) Prove that every continuous function on a closed interval is bounded. 6.5

(b) Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $]0, \infty[$. 6.5

(c) Define an open set and prove that the union of an arbitrary family of open sets is an open set. 6.5



3. (a) Define a convergent sequence and a bounded sequence. Show that the sequence $< a_n >$ defined by :

$$a_n = (-1)^n, \ \forall n$$

is bounded but not convergent. 6.5

(b) Show that the sequence $< a_n >$ defined by :

$$a_1 = 1, \quad a_{n+1} = \sqrt{2 + a_n}, \ \forall \ n \geq 1$$

is bounded and monotonic. Also find $\lim_{n\to\infty} a_n$. 6.5

(c) State and prove Cauchy convergence criterion for sequences. 6.5

4. (a) Prove that every monotonically increasing and bounded above sequence converges. 6

(b) If $< a_n >$ and $< b_n >$ are sequences of real numbers such that :

$$\lim_{n\to\infty} a_n = a, \quad \lim_{n\to\infty} b_n = b$$

then prove that :

$$\lim_{n\to\infty} (a_n + b_n) = (a + b).$$ 6

(c) Show that :

$$\lim_{n\to\infty} \frac{2^{3n}}{3^{2n}} = 0.$$ 6