4. (*a*) Find the rational roots of the equation : 6½

$$x^4 - x^3 - 19x^2 + 49x - 30 = 0.$$

(*b*) Solve the equation :

$$27x^3 + 42x^2 - 28x - 8 = 0,$$

the roots being in G.P. 6½

(*c*) If α, β, γ, be the roots of the equation : 6½

$$x^3 + px^2 + qx + r = 0, \ (r \neq 0),$$

find the value of :

(*i*) $\Sigma(\beta + \gamma)^2$

(*ii*) $\Sigma\alpha^{-2}$.

5. (*a*) Let n be a positive integer. Prove that the congruence class $[a]_n$ has a multiplicative inverse in Z_n if and only if $(a, n) = 1$. 6½

30/5/17 Eve.

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **2002**

Unique Paper Code : **62351201** **GC-4**

Name of the Paper : **Algebra**

Name of the Course : **B.A. (Prog.) Discipline Course**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Prove that the set V of all ordered triples of real numbers of the form $(x, y, 0)$ under the operations $\oplus$ and $\odot$ defined by :

$$(x, y, 0) \oplus (x', y', 0) = (x + x', y + y', 0)$$

$$c \odot (x, y, 0) = (cx, cy, 0)$$

forms a vector space over **R**. 6

P.T.O.

(*b*) Let V be a vector space with operators $\oplus$ and $\odot$. Let W be a non-empty subset of V. Prove that W is a subspace of V if and only if the following conditions hold :

(*i*) If u, v are vectors in W, then $u \oplus v$ is in W

(*ii*) If c is any real number and u is any vector in W, then $c \odot u$ is in W. 6

(*c*) Define basis of a vector space V. Check whether the set $\{(3, 2, 2), (-1, 2, 1), (0, 1, 0)\}$ forms basis for $\mathbf{R}^3$? 6

2. (*a*) Reduce the matrix :

$$\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

to its normal form and hence determine its rank. 6½

(*b*) Verify that the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

satisfies its characteristic equation and hence obtain A^{-1}. 6½

(*c*) Solve the system of linear equations : 6½

$$x - 3y + z = -1$$

$$2x + y - 4z = -1$$

$$6x - 7y + 8z = 7.$$

3. (*a*) If

$$\cos\alpha + 2\cos\beta + 3\cos\gamma = 0 = \sin\alpha + 2\sin\beta + 3\sin\gamma,$$

prove that :

$$\cos 3\alpha + 8\cos 3\beta + 27\cos 3\gamma = 18\cos(\alpha + \beta + \gamma)$$

$$\sin 3\alpha + 8\sin 3\beta + 27\sin 3\gamma = 18\sin(\alpha + \beta + \gamma).$$ 6

(*b*) Prove that : 6

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + \tan^4\theta}.$$

(*c*) Sum the series :

$$\cos\theta\sin\theta + \cos^2\theta\sin 2\theta + \ldots\ldots + \cos^n\theta\sin n\theta,$$

where $\theta \neq k\pi$. 6

(*b*) Prove that the set S = {0, 2, 4} is a subring of the ring Z_6 of integers modulo 6. 6

(*c*) Prove that the rigid motions of an equilateral triangle yields the group S_3. 6

(*b*) Consider the following permutations in S_7 : 6½

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 6 & 1 & 7 \end{pmatrix}$$

and $$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 7 & 4 & 6 & 3 \end{pmatrix}$$

Compute the following products :

(*i*) $\tau^{-1}\sigma\tau$

(*ii*) $\tau^2\sigma$.

(*c*) Prove that the set Q^+ of all positive rational numbers is an abelian group under the binary operation * defined by $a * b = \frac{ab}{3}$. 6½

6. (*a*) Let $G = GL_2(R)$. Prove that :

$$D = \left\{ \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix};\ ad \neq 0 \right\}$$

is a subgroup of G. 6