6. (a) Define multiple regression and polynomial regression model. Explain the role of orthogonal polynomials in fitting polynomial models in one variable. 7

(b) Write a note on the extra sum of squares method that can be used to test the hypotheses about any subset of regressor variables. 5.5

7. (a) For a simple linear regression model, develop a test for lack of fit. 5

(b) Suppose X_i, Y_i, Z_i, $i = 1,2,..n$ are $3n$ independent observations with common variance σ^2 and expectations $E(X_i)= \theta_1$, $E(Y_i)=\theta_2$ $E(Z_i) = \theta_1-\theta_2$, $i=1, 2, .., n$. Find the BLUEs of θ_1, θ_2 and $\theta_1+\theta_2$. Also find $\text{cov}\left(\hat{\theta}_1+\hat{\theta}_2\right)$. 7.5

8. Write notes on any **two** of the following : 6,6.5

(i) General linear model

(ii) Orthogonal columns in X matrix

(iii) Stepwise regression method

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **2279** **IC**

Unique Paper Code : 32371402

Name of the Course : **B.Sc. (Hons.) Staistics**

Name of the Paper : Linear Models

Semester : IV

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt **six** questions in **all**, selecting **three** questions from each **Section**.

Section - I

1. Suppose $\underset{\sim}{Y} = (Y_1, Y_2, ...Y_n)'$ to be a vector of n independent standard normal variates . Prove that a necessary and sufficient condition for $\underset{\sim}{Y}'A\underset{\sim}{Y}$ to be distributed as chi-square variate with k d.f. is that A an idempotent matrix of rank k. 12.5

2. (a) Let $X_1, X_2, ..., X_n$ be real numbers with mean $\bar{x}$. Consider the linear model $Y_i = \alpha+\beta(x_1 - \bar{x}) + \varepsilon_i$, $i = 1, 2, ..., n$ with the usual assumptions. Show that the BLUEs of α and β are uncorrelated. 5

P.T.O.

(b) Consider three independent ransom variables Y_1, Y_2 and Y_3 having common variance σ^2 and expectations as given below:

$$E(Y_1)=\beta_1+\beta_3, E(Y_2)=\beta_1+\beta_2 \; E(Y_3)=\beta_1+\beta_3,$$

Determine the condition of estimability of a parametric function. Also, determine the sum of squares due to error. 7.5

3. (a) Complete the following table for analysis of variance of a fixed effects two way classified data with one observation per cell : 6

Source of variation	Sum of Squares	Degrees of freedom	Mean square	Variance ratio
Factor A	26.8	4	..	..
Factor B	..	3	..	..
Error	..	...	2.5	
Total	85.3	...	....	

(b) Suppose $\underset{\sim}{Y} \sim N_3\left(\underset{\sim}{0}, I\right)$ and

$$A=\frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}, B=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

(i) Are $\underset{\sim}{Y}'A\underset{\sim}{Y}$ and $2y_1 + y_2$ independent ?

(ii) Are $\underset{\sim}{Y}'A\underset{\sim}{Y}$ and $B\underset{\sim}{Y}$ independent ?

(iii) Find the distribution of $\underset{\sim}{Y}'A\underset{\sim}{Y}$, stating the appropriate theorem to be used and also, find the distriibution of $\underset{\sim}{Y}'D\underset{\sim}{Y}$ and $\underset{\sim}{Y}/\underset{\sim}{Y}$, where D = I -A. Are $\underset{\sim}{Y}'A\underset{\sim}{Y}$ and $\underset{\sim}{Y}'D\underset{\sim}{Y}$ independent ? 6.5

4. Derive the Analysis of Covariance for a single factor with one covariate. Also, obtain the standard error of the difference between any two adjusted means. 12.5

Section - II

5. (a) Stating clearly the underlying assumptions of the simple linear regression model through the origin, obtain the least estimate of the regression parameter along with its variance. 5

(b) Suppose that we have fit the straight-line model $\hat{y}=\hat{\beta}_0+\hat{\beta}_1 x_1$ but the response is affected by a second variable x_2 such that the true regression function is $E(y)\,\beta_0+\beta_1 x_1+\beta_2 x_2$.

(i) Is the least-squares estimator of the slope in the original simple linear regression model unbiased ?

(ii) Show the bias is $\hat{\beta}_1$. 7.5