(*d*) Determine the solution of the initial boundary-value problem : 7

$$\begin{aligned} &u_{tt} = c^2 u_{xx}, && x > 0, \ t > 0, \\ &u(x,0) = f(x), && x \ge 0, \\ &u_t(x,0) = g(x), && x \ge 0, \\ &u_x(0,t) = q(t), && t \ge 0. \end{aligned}$$

Section-IV

5. Attempt any *two* parts out of the following :

(*a*) Determine the solution of the initial boundary-value problem by the method of separation of variables : 8

$$\begin{aligned} &u_{tt} = c^2 u_{xx}, && 0 < x < 1, \ t > 0, \\ &u(x, 0) = x(1-x), && 0 \le x \le 1, \\ &u_t(x,0) = 0, && 0 \le x \le 1, \\ &u(0, t) = u(1, t) = 0, && t \ge 0. \end{aligned}$$

(*b*) Prove the uniqueness of the solution of the problem : 8

$$\begin{aligned} &u_t = k u_{xx}, && 0 < x < l, \ t > 0, \\ &u(x,0) = f(x), && 0 \le x \le l, \\ &u_x(0,t) = u_x(l,t) = 0, && t \ge 0. \end{aligned}$$

(*c*) Determine the solution of the initial-boundary value problem : 8

$$\begin{aligned} &u_t = k u_{xx}, && 0 < x < \pi, \ t > 0, \\ &u(x,0) = \sin^2 x, && 0 \le x \le \pi, \\ &u(0,t) = u(\pi,t) = 0, && t \ge 0. \end{aligned}$$

This question paper contains 4 printed pages]

Roll No.

S. No. of Question Paper : **2250**

Unique Paper Code : **32351401** **IC**

Name of the Paper : **Partial Differential Equations**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Section-I

1. Attempt any *three* parts out of the following :

(*a*) Determine the integral surfaces of the equation :

$$x(y^2 + u)u_x - y(x^2 + u)u_y = (x^2 - y^2)u$$

with the data $x + y = 0, \ u = 1$. 6

(*b*) Apply the method of separation of variables to solve the initial-value problem :

$$x^2 u_{xy} + 9y^2 u = 0, \quad u(x,0) = \exp\left(\frac{1}{x}\right).$$ 6

(*c*) Reduce the following equation into canonical form and find the general solution :

$$u_x + u_y = u.$$ 6

(*d*) Solve the initial-value problem :

$$u_t + uu_x = 0$$

with the initial curve $x = \frac{\tau^2}{2}$, $t = \tau$, $u = \tau$. 6

Section-II

2. Attempt any *one* part out of the following :

(*a*) Show that the equation of motion of a vibrating string is :

$$u_{tt} = c^2 u_{xx}, \text{ where } c^2 = T/\rho.$$ 6

(*b*) Derive the wave equation of a string :

$$u_{tt} + au_t + bu = c^2 u_{xx},$$

where the damping force is proportional to the velocity, the restoring force is proportional to the displacement of a string, and a and b are constants. 6

3. Attempt any *two* parts out of the following :

(*a*) Determine the general solution of the equation :

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$$

by reducing it into canonical form. 7

(*b*) Transform the equation to the form $v_{\xi\eta} = cv$, c = constant,

$$u_{xx} - u_{yy} + 3u_x - 2u_y + u = 0$$

by introducing the new variable $v = ue^{-(a\xi + b\eta)}$, where a and b are undetermined coefficients. 7

(*c*) Classify the equation and reduce it to canonical form :

$$y^2 u_{xx} + 2xyu_{xy} + 2x^2 u_{yy} + xu_x = 0.$$ 7

Section-III

4. Attempt any *three* parts out of the following :

(*a*) Determine the solution of the initial boundary-value problem : 7

$$\begin{aligned} u_{tt} &= 4u_{xx}, & 0 < x < \infty,\ t > 0, \\ u(x,0) &= x^4, & 0 \le x < \infty, \\ u_t(x,0) &= 0, & 0 \le x < \infty, \\ u(0,t) &= 0, & t \ge 0. \end{aligned}$$

(*b*) Find the solution of the initial boundary-value problem : 7

$$\begin{aligned} u_{tt} &= u_{xx} & 0 < x < 2,\ t > 0, \\ u(x,0) &= \sin(\pi x/2), & 0 \le x \le 2, \\ u_t(x,0) &= 0, & 0 \le x \le 2, \\ u(0,t) &= 0,\ u(2,t) = 0,\ t \ge 0. \end{aligned}$$

(*c*) Determine the solution of the Goursat problem : 7

$$\begin{aligned} u_{tt} &= c^2 u_{xx}, \\ u(x,t) &= f(x) \text{ on } x - ct = 0 \\ u(x,t) &= g(x) \text{ on } t = t(x), \end{aligned}$$

where $f(0) = g(0)$.