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Your Roll No.....



Sr. No. of Question Paper : 5991

Unique Paper Code : 2372011103

Name of the Paper : Calculus

Name of the Course : B.Sc. (Hons) Statistics (NEP)

Semester : I

Duration : 3 Hours

Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all. **All** questions/parts carry equal marks.

1. a) If  $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x+\sqrt{y}}}\right)$  show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{4} \sin 2u$   
b) Find the values of  $a$  and  $b$  in order that  $\lim_{x \rightarrow 0} \frac{x(1+a \cos x) - b \sin x}{x^3}$  may be equal to 1.
2. a) If  $y = \cos(m \sin^{-1} x)$  show that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$  and hence find  $y_n(0)$ .  
b) If a function  $f$  is defined by

$$f(x) = \begin{cases} \frac{xe^{1/x}}{1+e^{1/x}}, & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Show that  $f$  is continuous but not derivable at  $x = 0$ .

3. a) Solve  $\int_0^1 \int_0^{x^2} e^{y/x} dx dy$ .  
b) Solve the differential equation  $(1 + y^2)dx = (\tan^{-1} y - x)dy$ .
4. a) Show that  $\int_0^{\pi/2} \tan^n \theta d\theta = \frac{1}{2} \pi \sec\left(\frac{n\pi}{2}\right)$ .  
b) Solve  $(x - y)^2 \frac{dy}{dx} = a^2$ .

5. a) Change the order of integration in  $\int_0^a \int_{\sqrt{a^2-x^2}}^{x-2a} \phi(x, y) dx dy$ .  
b) Assuming the validity of differentiation under integral sign, show that  $\int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)} dx = \frac{\pi}{2} \log(1+a)$ , if  $a \geq 0$ .
6. a) Solve  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$  with  $y = 0, x = 0$  and  $\frac{dy}{dx} = 0$ .  
b) Evaluate  $\lim_{x \rightarrow 0} \left[ \tan\left(\frac{\pi}{4} + x\right) \right]^{1/x}$ .
7. a) Solve  $D^3 - 3D^2D' + 4D'^3 = e^{x+2y}$ .  
b) If  $x^x y^y z^z = c$  show that at  $x = y = z$ ,  $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$ .
8. a) Show that the differential equation  $(x^4 - 2xy^2 + y^4)dx - (2x^2y - 4xy^3 + \sin y) dy = 0$  is exact and also solve it.  
b) If  $y = x \log[(ax)^{-1} + a^{-1}]$ , prove that  $x(x+1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = y - 1$ .