

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 13275
Unique Paper Code : 2354000006
Name of the Paper : Numerical Methods
Name of the Course : **GE**
Semester : V
Duration : 3 Hours



Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of non-programmable scientific calculator is allowed.

1) Define order of convergence.

Add the following numbers from left to right and right to left, rounding to 3 digits at each step:

$$8 + 0.6666 + 0.4444 + 0.2222$$

Find the relative error in each case and the number of significant digits to which the two values approximate the exact solution.

b) Determine the number of significant digits in the following numbers:

(i) 269007 (ii) 0.000045 (iii) 8.00293 (iv) 627900 (v) 5.8×10^{-9}

c) If the true value of a number $x = 1.6180339$ and its approximate value is given by 1.618, find the absolute, relative and percentage errors.

2)

a) Find a root of the equation $f(x) = 3x^3 - x^2 - 18x + 6$ in the interval $[0,1]$, using Bisection Method. Perform 5 iterations. Write the order of Convergence of Bisection Method.

b) Find a root of the equation $f(x) = 2^x - x^3$ taking $x_0 = 1$ and $x_1 = 1.5$, using Secant Method. Perform 5 iterations. Write the order of Convergence of Secant Method.

c) Find a root of the equation $f(x) = (35x^4 - 30x^2 + 3)/8$ taking $x_0 = 0.5$ and $x_1 = 1$, using Regula-Falsi Method. Perform 5 iterations. Write the order of Convergence of Regula-Falsi Method.

3) a) Solve the system of equations

$$2x + 10y - 5z = -53$$

$$10x - 2y + 3z = 23$$

$$3x - 4y + 10z = 33$$

using the Gauss elimination with partial pivoting.

P.T.O.

b) Solve the following system of linear equations using Gauss-Jacobi method, with $X^{(0)} = \{0,0,0\}$. Perform three iterations.

$$\begin{aligned} 4x + 2y + z &= 14 \\ 6x + 15y + 2z &= 72 \\ x + y + 54z &= 110 \end{aligned}$$

c) Solve the following system of equations using the Gauss-Seidel iteration method. Take $X^{(0)} = \{0,0,0\}$ and perform three iterations.

$$\begin{aligned} -8x_1 + x_2 - 2x_3 &= -20 \\ 2x_1 - 6x_2 - x_3 &= -38 \\ -3x_1 - x_2 + 7x_3 &= -34 \end{aligned}$$

4. (a). Define the interpolating polynomial, and using the Lagrange formula, find the polynomial that passes through the points (1,2), (2,3), (4,1).

(b). Prove the following relations:

(i) $(1 + \Delta)(1 - \nabla) = 1$

(ii) $\Delta - \nabla = \nabla\Delta$

where each symbol has its usual meaning.

(c). Show that the third divided difference with arguments a,b,c,d of the function $f(x) = \frac{1}{x}$ is $\frac{-1}{abcd}$.

5. (a). Use the three-point backward difference formula

$$f'(x_i) \approx \frac{3f(x_i) - 4f(x_i - h) + f(x_i - 2h)}{2h}$$

to approximate the derivative of $f(x) = \sin x + \cos x$ at $x_i = \frac{\pi}{2}$, taking $h = 1, 0.1$ and 0.01 .

(b). For the following data, find $f''(7.50)$ by using the formula

$$f''(x_i) \approx \frac{f(x_i) - 2f(x_i+h) + f(x_i+2h)}{h^2}$$

x	7.47	7.48	7.49	7.50	7.51	7.52	7.53
$f(x)$.193	.195	.198	.201	.203	.206	.208

(c). Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by using Trapezoidal rule. Hence obtain the approximate value of π .

6. (a). Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with the initial condition $x = 0, y = 1$. Find $y(0.1)$ using Euler's method with step size $h = 0.025$.

(b). Using mid-point (Runge Kutta second order) method, solve the initial value problem $\frac{dy}{dx} = x^2 + \sqrt{y}$ from $x = 0$ to $x = 0.6$ where $y(0) = 1$ by using $h = 0.2$.

(c). Apply Runge Kutta method of fourth order to find the approximate value of y for $x = 0.2$, in steps of 0.1, if $\frac{dy}{dx} = x + y^2$, given that $y = 1$ where $x = 0$.