

[This question paper contains 2 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 12910 **K**

Unique Paper Code : 2354001001

Name of the Paper : GE Fundamentals of Calculus

Name of the Course : **Common Prog. Group**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory and carry equal marks.
3. This question paper has six questions.
4. Attempt any two parts from each question.

1. a) (i) Show that $\lim_{x \rightarrow 0} \frac{x - |x|}{x}$ does not exist.

(ii) Evaluate $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$

b) Examine the continuity at $x = 1$ and $x = 2$ of the function $f(x)$ defined below

$$f(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 2 - x, & 1 < x \leq 2 \\ x^2 - 2x, & x > 2 \end{cases}$$

Also, the type of discontinuity, if any.

c) Discuss the differentiability at $x = 0$ of the function $f(x)$ defined below

$$f(x) = x \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, \quad x \neq 0 \text{ and } f(0) = 0.$$

2. a) Find the n^{th} differential coefficient of $\tan^{-1}\{2x/(1-x^2)\}$.

b) If $y = \cos(m \sin^{-1}x)$, evaluate $y_n(0)$.

c) If $u = \sin^{-1} \left\{ \frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}} \right\}$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{144} \tan u (13 + \tan^2 u).$$

3. a) State Rolle's theorem and verify the same for the function

$$f(x) = (x-a)^2(x-b)^3 \text{ for all } x \in [a, b].$$



- b) State and prove Lagrange's mean value theorem.
- c) Verify the Cauchy's mean value theorem for the following pair of functions:
 (i) $f(x) = \sin x$ and $g(x) = \cos x$ in the domain $[-\pi/2, 0]$.

(ii) $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ in the domain $[1, 5]$.

4. a) State Taylor's theorem and prove that

$$\sin ax = ax - \frac{(ax)^3}{3!} + \frac{(ax)^5}{5!} - \dots + \frac{(ax)^{n-1}}{(n-1)!} \sin\left(\frac{n-1}{2}\pi\right) + \frac{(ax)^n}{n!} \sin\left(a\theta x + \frac{n\pi}{2}\right),$$

where a is a nonzero real number and $0 < \theta < 1$.

- b) Find the Maclaurin's series of $\log(1+x) \forall -1 < x \leq 1$, with Cauchy's form of the remainder.

- c) Evaluate the following:

(i) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \cot^2 x \right)$

(ii) $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$

5. a) Trace the curve

$$r = 2 + 3\cos\theta, \quad 0 \leq \theta \leq 2\pi$$

- b) Sketch the graph of the function

$$y = x^3 - 3x + 2$$

- c) Find the oblique asymptotes of the curve

$$y^3 - x^2y - 2xy^2 + 2x^3 - 7xy + 3y^2 + 2x^2 + 2x + 2y + 1 = 0$$

6. a) Determine the intervals of concavity and points of inflexion of the function

$$f(x) = x^3 - 3x^2 + 1.$$

- b) Use the second derivative test to find the points of local maxima and minima of the function $f(x) = 12x^5 - 45x^4 + 40x^3 + 6$. Also determine the intervals on which $f(x)$ is increasing and decreasing.

- c) Determine the critical points of the function $f(x) = \frac{x+1}{x^2+3}$. Also, find the horizontal and vertical asymptotes.