[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 12634 **K**

Unique Paper Code : 2354002001

Name of the Paper : Differential Equations

Name of the Course : **COMMON PROG GROUPS (Generic Elective)**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **Both** parts of the question to be attempted together.
4. **All** questions carry equal marks.
5. The use of a calculator is not allowed.

1 (a) Show that $y = 4e^{2x} + 2e^{-3x}$ is a solution of the initial-value problem (7.5)

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0, \qquad y(0) = 6, \qquad y'(0) = 2.$$

Is $y = 2e^{2x} + 4e^{-3x}$ also a solution of this problem? Explain why or why not.

(b) Solve the following initial value problem (7.5)

$$xy\frac{dy}{dx} + 4x^2 + y^2 = 0, \qquad y(2) = -7, \qquad x > 0.$$

(c) Solve the differential equation (7.5)

$$\left(\frac{2s-1}{t}\right)ds + \left(\frac{s-s^2}{t^2}\right)dt = 0.$$

2 (a) Solve the differential equation (7.5)

$$x\,dy + (xy + y - 1)\,dx = 0.$$

(b) Solve the differential equation (7.5)

$$(2xy^2 + y)\,dx + (2y^3 - x)\,dy = 0$$

by first finding an integrating factor.

(c) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = c^2$. (7.5)

3 (a) Solve the initial value problem (7.5)

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0; \; y(0) = -3, \qquad y'(0) = -1$$

(b) Find the general solution of the following differential equation by using the method of undetermined coefficients. (7.5)

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5\,y = 4\,e^{2\,x}.$$

(c) Use the method of variation of parameters to find the general solution of the differential equation (7.5)

$$\frac{d^2y}{dx^2} - 2\,\frac{dy}{dx} + y = \,e^x \sin x.$$

4 (a) Find the general solution of the differential equation (7.5)

$$x^3\frac{d^3y}{dx^3} - x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y$$
$$= x^3.$$

(b) Transform the following fourth-order ordinary differential equation into a system of four first-order ordinary differential equation (7.5)

$$\frac{d^4y}{dx^4} + 4\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + \,y = 0.$$

(c) Solve the linear system (7.5)

$$\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t$$

$$\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}.$$

5 (a) Find the general solution of the following differential equation (7.5)

$$x^2(y\, -\, u)u_x + y^2\,(u\, -\, x)u_y = u^2(x - y)$$

(b) Find the solution of the equation (7.5)

$$x\,u_x + y\,u_y = x\,e^{-u}$$

with Cauchy data $u = 0$ on $y = x^2$.

(c) Reduce the following equation into canonical form and find the general solution (7.5)

$$u_x\, +\, u_y\, =\, y.$$

6 (a) Solve the following initial value problem using the method of separation of variables (7.5)

$$u_x + 2u_y = 0, \qquad u(0, y) = 4e^{-2y}.$$

(b) Find the solution of the initial value systems (7.5)

$$u_t\, -\, 2uu_x\, =\, v\, -\, x,\; v_t + av_x = 0$$

with $u(x, 0)\, =\, x$ and $v(x, 0)\, =\, x$.

(c) Obtain the general solution of (7.5)

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y\, = 2.$$

(2000)