[This question paper contains 2 printed pages.]

Your Roll No..............

**Sr. No. of Question Paper : 12337** **K**

Unique Paper Code : 2354000015

Name of the Paper : Topics In Multivariate Calculus

Name of the Course : **Common Prog Group**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator is not allowed.

1. (a) Define $\epsilon - \delta$ definition of limit of a function of two variables. Using this definition, show that $\lim_{(x,y)\to(0,0)} (x^2+y^2)^{\frac{3}{2}} = 0$.

(b) (i) Given that the function $f(x,y) = \begin{cases} \frac{3x^3-3y^3}{x^2-y^2} & for\ x^2 \neq y^2 \\ A & otherwise \end{cases}$ is continuous at the origin, what is $A$?

(ii) Let $z$ be defined implicitly as a function of $x$ and $y$ by the equation $x^2z + yz^3 = x$. Determine $z_x$ and $z_y$.

(c) When two resistances $R_1$ and $R_2$ are connected in parallel, the total resistance $R$ satisfies

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If $R_1$ is measured as 300 ohms with a maximum error of 2% and $R_2$ is measured as 500 ohms with a maximum error of 3%, what is the maximum percentage error in $R$?

2.(a) (i) Define gradient of a function of two variables.

(ii) In what direction, is the function defined by $f(x,y) = xe^{2y-x}$ increasing, rapidly at the point $P_0(2,1)$, and what is the maximum rate of increase? In what direction is $f$ decreasing most rapidly?

(b) (i) Find a vector that is normal to the level surface $x^2 + 2xy - yz + 3z^2 = 7$ at the point $P_0(1,1,-1)$.

(ii) Let $z = f(x,y)$, where $x = at$ and $y = at$ for constants $a$ and $b$. Assuming all necessary differentiability, find $\frac{d^2z}{dt^2}$ in terms of the partial derivatives of $z$.

(c ) Find the point on the plane $x + 2y + z = 5$ that is closest to the point $P(0,3,4)$.

3. (a) (i) Evaluate $\iint_R \frac{2xy}{x^2+1} dA$ ; $R: 0 \leq x \leq 1, 1 \leq y \leq 3$ using iterated integration.

(ii) Find the volume of the solid bounded below by the rectangle $R$ in the $xy$ plane and above by the graph of $z = f(x,y)$ where $f(x,y) = 2x + 3y$ ; R: $0 \leq x \leq 1,\ 0 \leq y \leq 2$.

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(b) Evaluate $\iint_D x\,dy\,dx$ ; $D$ is the region between the parabola $y = x^2$ and the line $y = 2x + 3$.

(c ) Evaluate $\int_0^2 \int_y^{\sqrt{4-y^2}} e^{x^2+y^2} dxdy$ by converting to polar coordinates

4. (a) Find the volume of the tetrahedron $T$ bounded by the plane $x + y + z = 1$ and the coordinate planes $x = 0,\ y = 0,\ z = 0$.

(b) Use cylindrical coordinates to evaluate $\iiint_D z(x^2 + y^2)^{-1/2} dx\,dy\,dz$ where $D$ is solid bounded above by the plane z = 2 and below by the surface $2z = x^2 + y^2$

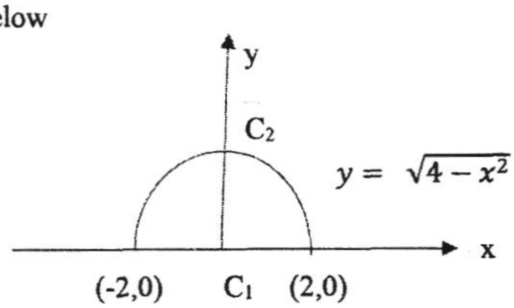
(c ) Let $D$ be the region in $xy -$ plane that is bounded by coordinate axes and the line $x + y = 1$. Use suitable change of variables $u = x - y, v = x + y$ to compute the integral $\iint_D \frac{(x-y)^6}{(x+y)^4} dy\,dx$.

5. (a) Evaluate the line integral $\int_C (y\,dx + x\,dy + zdz)$, where $C$ is the helical path given by

$x = \cos t,\ y = \sin t,\ z = t$ for $0 \le t \le \pi/2$.

(b) Verify the vector field $\boldsymbol{F}(x, y) = (x + 2y)\,\boldsymbol{i} + (2x + y)\,\boldsymbol{j}$ is conservative and find a scalar potential f for **F**. Then evaluate the line integral $\int_C \mathbf{F}.d\mathbf{R}$, where $C$ is any smooth path connecting $A(0,0)$ to $B(1,1)$.

(c ) Verify Green's Theorem for the line integral $\oint_C (2ydx - xdy)$ where $C$ is the closed path as shown in the figure below




Path C

6. (a) Evaluate $\iint_S (x^2 + y^2) dS$ where $S$ is the surface of the hemisphere $z = \sqrt{1 - x^2 - y^2}$.

(b) Use Stokes' theorem to evaluate $\iint_S (\text{curl } \mathbf{F}.\mathbf{N}) dS$, where $\mathbf{F} = x\,\boldsymbol{i} + y^2\boldsymbol{j} + xyz\,\boldsymbol{k}$ and $S$ is that part of paraboloid $z = 4 - x^2 - y^2$ with $z \ge 0$. Use the upward unit normal vector.

(c ) Use the divergence theorem to evaluate the surface integral $\iint_S \mathbf{F}.\mathbf{N}\,dS$ for $\boldsymbol{F} = 2y\,\boldsymbol{i} - z\,\boldsymbol{j} + 3x\,\boldsymbol{k}$ and $S$ is the surface of the upper five faces of the unit cube $0 \le x \le 1,\ 0 \le y \le 1,\ \ 0 \le z \le 1$ missing $z = 0$.