[This question paper contains 4 printed pages.]

Your Roll No.....

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Sr. No. of Question Paper : **5853**

Unique Paper Code : 2372013503 (NEP-UGCF)

Name of the Paper : Stochastic Processes

Name of the Course : **B.Sc (Hons.)**

Semester : V

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **TWO** sections in this paper.
3. **All** sections are compulsory.
4. Use of nonprogrammable scientific calculator is allowed.

Section I

Attempt any **five** parts :

1a) Let X_n, for n even, takes values $+1$ and -1, each with probability $\frac{1}{2}$, and for n odd, take values $\sqrt{a}, \frac{-1}{\sqrt{a}}$, with probabilities $\frac{1}{1+a}$, $\frac{a}{1+a}$ respectively (a is real number >-1 and $\neq 0,1$). Further let X_n's be independent. Is $\{X_n, n \geq 1\}$ covariance stationary?

b) In a series of Bernoulli trials with probability of success p, let X denote the number of failures preceding the first success and Y, the number of failures following the first success and preceeding the second success, then show that the p.g.f. of Z=X+Y is $P(s,s)=\left(\frac{p}{1-qs}\right)^2$. Hence obtain an expression for $P(Z = r)$.

c) Suppose a ball is thrown at random to one of the r cells. Let X_n ($n \geq 1$) be said to be in state k(=1,2,…,r) if after n throws k cells are occupied. Obtain the transition probability matrix of $\{X_n, n \geq 1\}$.

d) In the classical ruin problem, prove that a fair game remains fair and no unfair game can be changed to unfair game.

P.T.O.

e) Prove that the interval between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ having parameter λ has a negative exponential distribution with mean $\frac{1}{\lambda}$.

f) Let X be a non-negative integral valued random variable with probability $P(X = n) = p_n$, $n = 0,1,2...$, and probability generating function $P(s) = \sum_{k=0}^{\infty} p_k s^k$. Find the generating function for $P(X > n)$.

g) Construct the transition probability matrix when game starts with \$6 and gambler with initial capital \$3 starts the game and his probabilities of winning, losing and drawing a game are (0.46, 0.44,0.10) respectively and stopping criteria is either a gambler is ruined or wins all the capital at stake. Construct the corresponding transition probability matrix Further, find expected duration of the game when gambler either wins or loses a game with equal probability and there is no possibility of draw. $(3 \times 5 = 15)$

Section II

Attempt any **five** questions:

2a) Let a_n denote the number of ways in which score n can be obtained by throwing a die any number of times. Show that the g.f. of $\{a_n\}$ is given by $\left\{1 - s - s^2 - s^3 - s^4 - s^5 - s^6\right\}^{-1} - 1$.

b) Let $S_N = X_1 + X_2 + ... + X_N$, where X_i's are i.i.d. random variables and N is a random variable independent of X_i's. Show that

$$Var(S_N) = E(N)Var(X_i) + Var(N)(E(X_i))^2$$

Also show that for a compound geometric distribution

$$\frac{Var(S_N)}{(E(S_N))^2} \geq k \frac{Var(X_i)}{(E(X_i))^2}$$

where k is the compounding parameter. (8,7)

3a) Let X be a zero- one truncated Poisson variate having the p.m.f.:

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!(1 - e^{-\lambda} - \lambda e^{-\lambda})}, k = 2,3,.....$$

Obtain the probability generating function of r.v. X and hence obtain mean and variance of X. Further, if X_1 and X_2 are i.i.d. having p.m.f. same as that of r.v. X, then find the probability generating function of $X_1 + X_2$.

b) Let $S_N = X_1 + X_2 + ... + X_N$, where N has Poisson distribution with mean a. If X_i's have i.i.d. binomial distributions with parameters m and p, then show that

i) The joint distribution of S_N and N has the probability mass function

$$P(N = n \cap S_N = y) = \frac{e^{-a} a^n (mn)!\, q^{mn-y} p^y}{n!\, y!\, (mn-y)!}$$

ii) $\text{Cov}(N, S_N) = map$ (8,7)

4a) On 1st January 2024 each of the three leading automobile companies Mahindra, Hyundai, Tata Motors launched a new model of electric car. When the new models were launched each company had an equal share of the market but during the year the following changes in the market share were observed. Mahindra retained 90% of its customers but lost 3% to Hyundai and 7% to Tata Motors. Hyundai retained 70% of its customers and lost 10% to Mahindra and 20% to Tata Motors. Tata Motors managed to retain 80% of its customers but lost 10% to Mahindra and 10% to Hyundai. Construct the corresponding transition probability matrix and identify the nature of states. Use Markov chain analysis to determine the market share of the three auto companies after two months and also predict the market share of each company in the long run.

b) i) Show that the initial distribution together with transition matrix completely defines a Markov chain.

ii) Find the nature of state 1, if

$$p^{(1)}{}_{11} = 0.352, p^{(2)}{}_{11} = 0.3775, p^{(3)}{}_{11} = 0.3975, p^{(4)}{}_{11} = 0.4115, p^{(5)}{}_{11} = p^{(6)}{}_{11} = p^{(7)}{}_{11} = \ldots = p^{(n)}{}_{11} = 0.4118 \text{ as } n \to \infty.$$ (8,7)

5a) Let $\{X_n, n \geq 0\}$ be a Markov chain with states $\{1,2,3,4,5\}$ and transition probability matrix:

$$P = \begin{pmatrix} 0.4 & 0.6 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.3 & 0.7 & 0.0 \\ 0.0 & 0.0 & 0.5 & 0.4 & 0.1 \\ 0.0 & 0.0 & 0.0 & 0.8 & 0.2 \end{pmatrix}$$

Identify the nature of all the states of the given Markov chain.

b) Show that in an irreducible Markov chain, all the states are of the same type i.e. the states are either all transient, all persistent null, or all persistent non-null. Further show that the persistence states are aperiodic or periodic with the same period.

(8,7)

6a) The births in hospital X in the city of Patna occur in accordance with Poisson process with parameter λ. The probability that an individual born is male is p and the probability of a female child born is q. Prove that the male births form a Poisson process with parameter λp and the female births form a Poisson process with parameter λq.

b) Describe the classical ruin problem. Obtain the probability of gambler's winning the game. (8,7)

7a) State the postulates of a Poisson process. Show that under these postulates N(t), the number of occurrences of an event E up to the epoch t follow a Poisson distribution with mean λt.

b) If $\{N_1(t); t \geq 0\}$ and $\{N_2(t), t \geq 0\}$are two independent Poisson processes with parameters λ_1 and λ_2 respectively, then

(i) Obtain the distribution of $N(t) = N_1(t) + N_2(t)$ and identify the same.

(ii) Find the auto correlation between $N(t)$and$N(t+s)$. (8,7)

(1000)